ସରକାରୀ ବହୁବୃତ୍ତି ବୈଷୟିକ ଶିକ୍ଷାନୁଷ୍ଠାନ କନ୍ଧମାଳ, ଫୁଲବାଣୀ

GOVERNMENT POLYTECHNIC KANDHAMAL, PHULBANI

DEPARTMENT OF MECHANICAL ENGINEERING

LECTURER NOTE BY - Dr DILLIP KUMAR PANIGRAHI

LECTURER IN MECHANICAL

SUBJECT NAME – Strength of Material

SECOND YEAR, 3^{rd} SEMESTER

SIMPLE STRESS AND STRAIN

STRESS

- It may be defined as the internal resisting force offered by the body due to the application of an external force.
- It is denoted by σ
- mathmatically

$$\text{stress } \sigma = \frac{\text{force}}{\text{cross section area}}$$

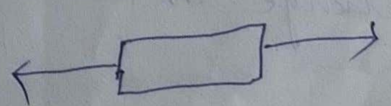
* It's SI unit is N/m^2

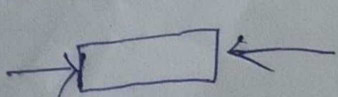
STRAIN

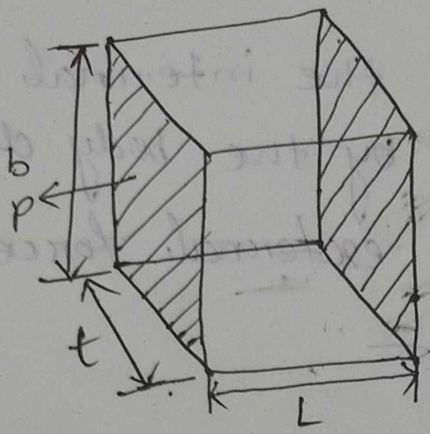
- It may be defined as the ratio between change in dimension to the original dimension
- It is denoted by e / ε → ep. silon
- mathmatically

$$\text{strain}, e = \frac{\text{change in dimension}}{\text{original dimension}}$$

- It has no unit.

force - ① Tensile force 

② compressive force 



$L + \delta L$

$b - \delta b$

$t - \delta t$

linear strain $= \frac{\delta l}{l}$

lateral strain $= \frac{\delta b}{b}, \frac{\delta t}{t}$

Poisson's Ratio

→ It may be difined as the ratio between lateral strain to the linear strain.

→ It is denoted by $\frac{1}{m}$ or μ

→ mathematically poisson's Ratio, $\left(\frac{1}{m} \text{ or } \mu\right)$

$$= \frac{\text{lateral strain}}{\text{linear strain}}$$

change in lenth $= \theta l$

change in breadth $= \theta b$

change in thickness $= \theta t$

HOOK'S law

It states that with in the elasticeiment the stress is directly proposenal to the strain within the elastic limit

$$\delta \propto e$$

$$\Rightarrow \sigma = Ee$$

E = proportionality constant known as young's moduluus or modulous of elasticisty.

Deformation of a body due to force acting on it.

stress $\delta = \frac{P}{A}$

strean $\varepsilon = \frac{\delta l}{l}$

Huck's law $\delta \propto E$

$$\Rightarrow \delta = \varepsilon \cdot e$$

$$\Rightarrow E = \frac{\delta}{E} = \frac{(P/A)}{E} = \frac{P}{AE}$$

$$\Rightarrow \frac{\delta l}{l} = \frac{P}{AE}$$

$$\Rightarrow \delta l = \frac{Pl}{AE}$$

where, P = force acting on the body

l = length of the body

A = cross section Area

E = young's modulus or modulus of Elasticity.

Ex :- 3.2

A hollow cylinder 2m long has an outside diameter of 50 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 KN, find the stress in the cylinder. Also find the deformation of the cylinder, if the value of modulus of elasticity for the cylinder material is 100 GPa.

Given

$l = 2m$

$D = 50 mm = 0.05 m$

$d = 30 mm = 0.03 m$

$P = 25 KN = 25 \times 10^3$

Stress

cross sectional Area. $A = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(0.05^2 - 0.03^2)$

$= 1.26 \times 10^{-3} m^2$

stress $\delta = \frac{P}{A} = \frac{25 \times 10^3}{1.26 \times 10^{-3}}$

$= 19.84 \times 10^{-6} N/m^2$

Deformation

$\delta = 100 GPa = 100 \times 10^9 Pa$

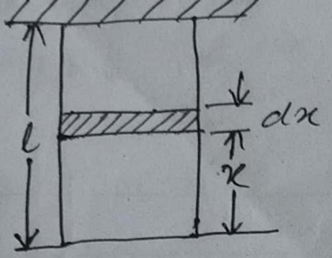
$= 100 \times 10^9 N/m^2$

$\delta L = \frac{Pl}{AE}$

$= \frac{25 \times 10^3 \times 2}{1.26 \times 100 \times 10^9}$

$= 3.968 \times 10^{-4} m$

Deformation of a body due to self weight :-



Let l = length of the bar

A = cross sectinal area of the bar

E = younge's modules of the bar meterial

w = specific weight of the bar meterial

specific weight = $\frac{\text{weight}}{\text{volume}}$

$\Rightarrow$ weight = specific weight $\times$ volume

weight of bar for a length of x

$$P = w \cdot Ax$$

diformation of small section of the bar due to weight of the bar of a lenght of x

$$\delta x = \frac{Pl}{AE} \quad \frac{(w \cdot Ax)\, dx}{A \cdot E}$$

Total diformation of the bar

$$dx = \delta l = \int_0^l \frac{wAx\, dx}{A \cdot E}$$

$$= \frac{w}{E}\left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{wl^2}{2E}$$

$$\delta l = \frac{wl^2}{2E}$$

$$w = \frac{W}{V} = \frac{W}{Al}$$

$$\Rightarrow W = w.Al$$

$$\delta L = \frac{wl^2}{2E}$$

$$= \frac{w.Al^2}{2AE}$$

$$= \frac{wAl.l}{2AE}$$

$$= \frac{W.L}{2AE}$$

$A = 2mm^2 = 2 \times 10^{-6}\ m^2$

$W = 12N. +$ own weight

$l = ?$

$\delta l = 0.6mm = 0.6 \times 10^{-3} m$

$E = 150 GPa = 150 \times 10^9 Pa$

$$\delta l = \frac{WL}{2AE}$$

$$\Rightarrow 0.6 \times 10^{-3} m = \frac{12 \times l}{2 \times 2 \times 10^{-6} \times 150 \times 10^9}$$

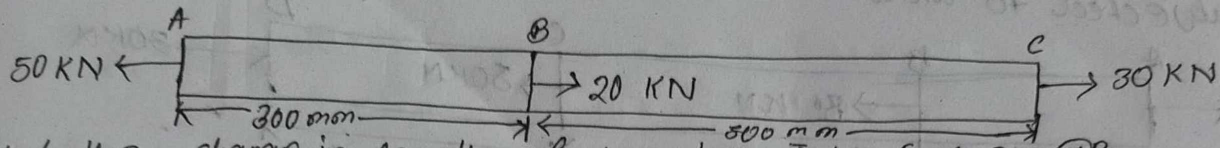
$$\Rightarrow l = \frac{0.6 \times 10^{-3} \times 2 \times 2 \times 10^{-6} \times 150 \times 10^9}{12}$$

$$= 30m$$

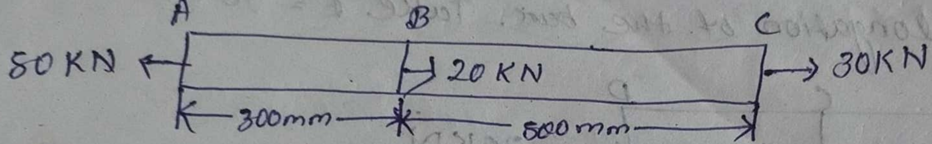
Principle of ~~superpostion~~ superposition :-

Ex:-2.10

A steel bar of cross-sectional area $200\ mm^2$ is loaded as shown in Fig 2.6. Find the change in length of the bar. Take E as 200GPa.



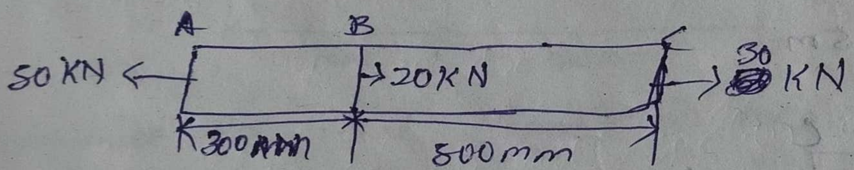
Find the change in length of the bar. Take E as 200 GPa.

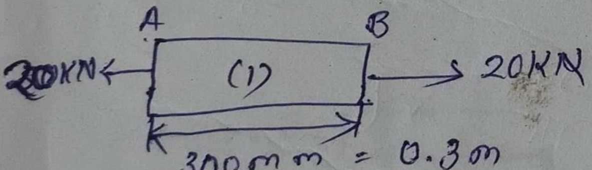


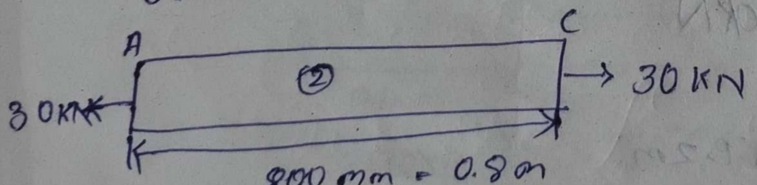
$A = 200\ mm^2 = 200 \times 10^{-6}\ m^2$

$\delta l = ?$

$E = 200GPa = 200 \times 10^9\ Pa$







$\delta l = \delta l_1 + \delta l_2$

$= \frac{P_1 l_1}{AE} + \frac{P_2 l_2}{AE}$

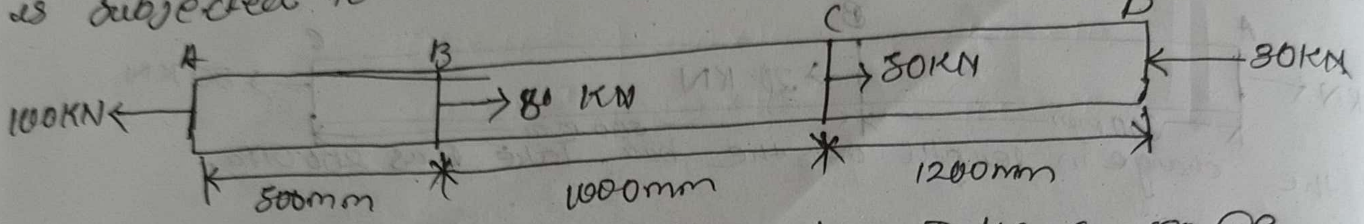
$= \frac{1}{AE}(P_1 l_1 + P_2 l_2)$

$$= \frac{1}{200\times10^{-6}\times200\times10^{9}}\left[\{20\times10^{3}\times0.3\}+\{30\times10^{3}\times0.8\}\right]$$

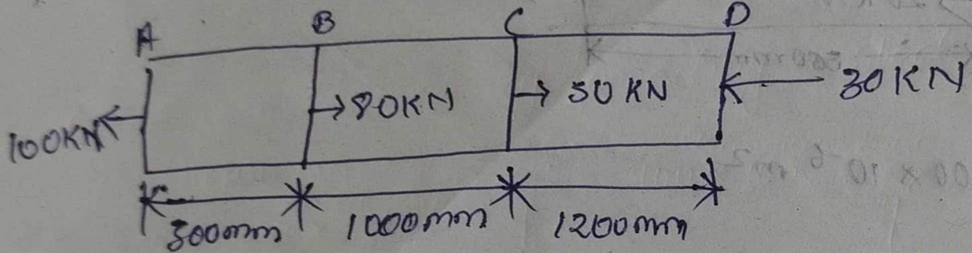
$$= 7.5\times10^{-4}\,m$$

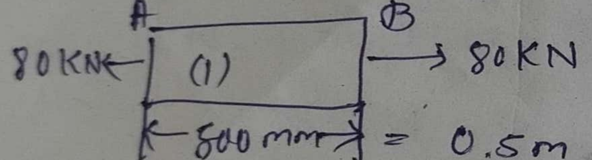
Ex: 2.11

A brass bar, having cross-sectional area of $800\,mm^2$ is subjected to axial forces as shown in Fig 2.8.

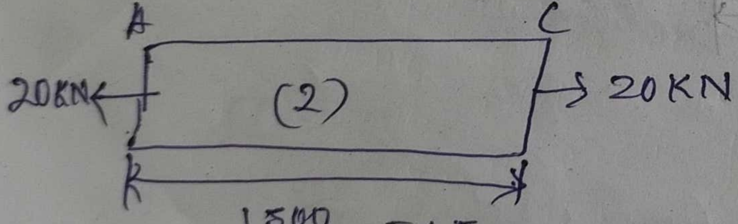


Find the total elongation of the bar. Take E = 80 GPa.

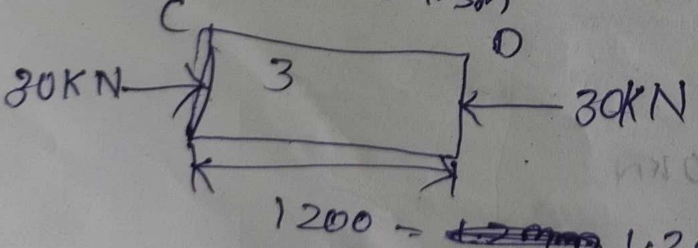




= 0.5m



1500 = 1.5m



1200 = ~~1.2mm~~ 1.2m

$$\delta l = \delta l_1 + \delta l_2 - \delta l_3$$

$$= \frac{P_1 l_1}{AE} + \frac{P_2 l_2}{AE} - \frac{P_3 l_3}{AE}$$

$$= \frac{1}{AE}\left[P_1 l_1 + P_2 l_2 - P_3 l_3\right]$$

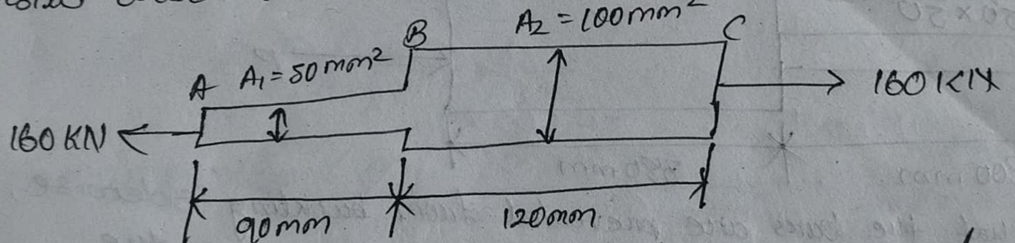
$$= \frac{1}{500\times10^{-6}\times30\times10^{9}}\left[\{80\times10^{3}\times0.5\}+\{20\times10^{3}\times1.5\}-\{30\times10^{3}\times1.2\}\right]$$

$$= 8.5\times10^{-4}$$

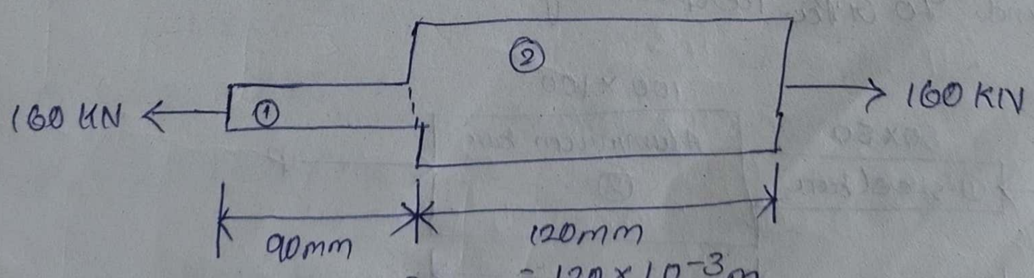
Stress in the bars of Different sections :-

Ex :- 3.1

An automobile component shown in fig 3.2 is subjected to a tensile load of 160 KN.



Determine the total elongation of the component, if its modules of elasticity is 200GPa.



$A_1 = 50\ mm^2 = 50\times10^{-6}\ m^2$

$A_2 = 100\ mm^2 = 100\times10^{-6}\ m^2$

$\delta l = ?$

$E = 200\ GPa = 200\times10^{9}\ Pa$
$= 200\times10^{9}\ N/m^2$

$$\delta l = \delta l_1 + \delta l_2$$

$$= \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E}$$

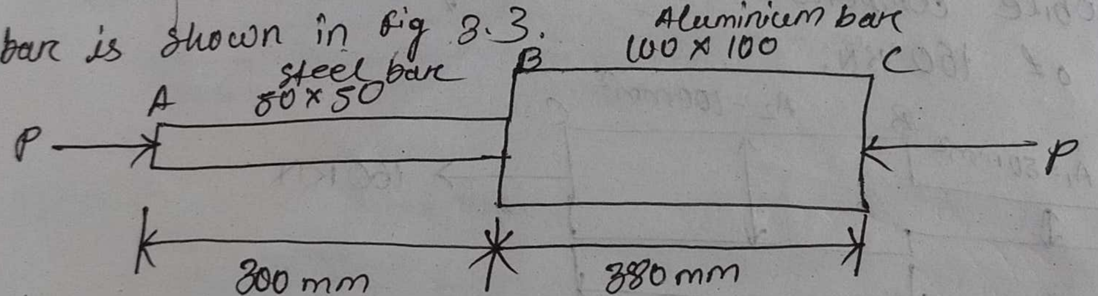
$$= \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2}\right]$$

$$= \frac{160 \times 10^3}{200 \times 10^9}\left[\frac{90 \times 10^{-3}}{50 \times 10^{-6}} + \frac{120 \times 10^{-3}}{100 \times 10^{-6}}\right]$$

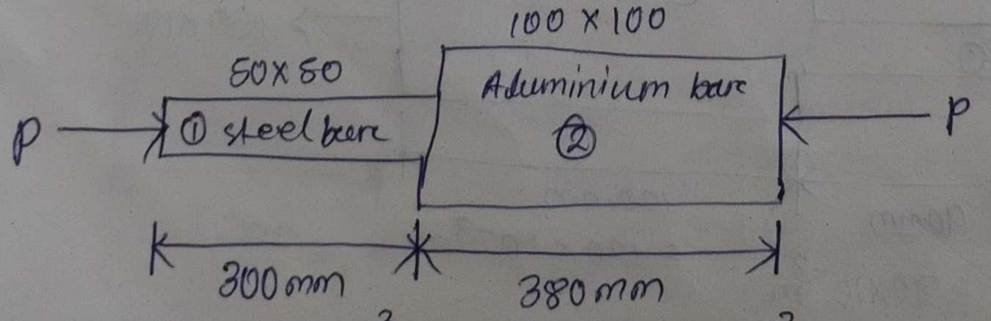
$$= 2.4 \times 10^{-3}\,m$$

Ex :- 3.2

A member formed by connecting a steel bar to an aluminium bar is shown in fig 3.3.



Assuming that the bars are prevented from buckling sidewise, calculate the magnitude of force P, that will cause the total length of the member to decrease by 0.25 mm. The values of elastic modulus for steel and aluminium are 210 GPa and 70 GPa respectively.



$= 300 \times 10^{-3}\,m$ $\quad$ $= 380 \times 10^{-3}\,m$

$A_1 = 50 \times 50 = 2500\, mm^2 = 2500 \times 10^{-6} m^2$

$l_1 = 300\, mm = 300 \times 10^{-3} m$

$A_2 = 100 \times 100 = 10000 \times 10^{-6}\, m^2$

$l_2 = 380\, mm = 380 \times 10^{-3} m$

$E_1 = 210\, GPa = 210 \times 10^9\, N/m^2$

$E_2 = 70\, GPa = 70 \times 10^9\, N/m^2$

$\delta l = 0.25\, mm$
$= 0.25 \times 10^{-3} m$

$P = ?$

$\delta l = \delta l_1 + \delta l_2$

$= \dfrac{P l_1}{A_1 E_1} + \dfrac{P l_2}{A_2 E_2}$

$\Rightarrow \delta l = P \left[\dfrac{l_1}{A_1 E_1} + \dfrac{l_2}{A_2 E_2} \right]$

$\Rightarrow 0.25 \times 10^{-3} = P \left[\dfrac{300 \times 10^{-3} m}{2500 \times 10^{-6} \times 210 \times 10^9} + \dfrac{380 \times 10^{-3}}{10000 \times 10^{-6} \times 70 \times 10^9} \right]$

$\Rightarrow 0.25 \times 10^{-3} = P\ 1.11 \times 10^{-9}$

$\Rightarrow P = \dfrac{0.25 \times 10^{-3}}{1.11 \times 10^{-9}}$

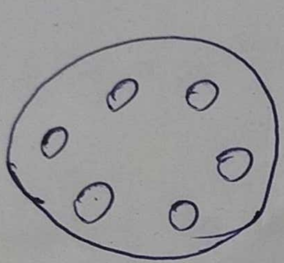
$\Rightarrow P = 225225\ N$

$\Rightarrow P = 225\ KN$

Stress in bars of composite section

Ex :- 3.15

A reinforced concrete circular section of 50,000 mm2 cross-sectional area carries 6 reinforcing bars whose total area is 500 mm2. Find the safe load, the column can ~~carry~~ carry, if the concrete is not to be stressed more than 3.5 MPa. Take modular ratio for steel and concrete as 18.



Reinforced concrete circular cross sectional area

$A_{Reinforced} = 50,000\ mm^2 = 50,000 \times 10^{-6}\ m^2$

No. of reinforcing bars = 6

$A_{steel} = 500\ mm^2 = 500 \times 10^{-6}\ m^2$

safe load, P = ? the column can carry

$\sigma_c = 3.5\ MPa = 3.5 \times 10^6\ Pa$

$= 3.5 \times 10^6\ N/m^2$

$A_{concrete} = 50,000 - 500 = 49,500\ mm^2$

$= 49,500 \times 10^{-6}\ m^2$

modular ratio of steel and concrete = 18

$\Rightarrow \dfrac{E_s}{E_c} = 18$

$\delta l_c = \delta l_s$

$\Rightarrow \dfrac{\delta l_c}{l_c} = \dfrac{\delta l_s}{l_s}$

$\Rightarrow e_c = e_s$

$\Rightarrow \dfrac{\sigma_c}{E_c} = \dfrac{\sigma_s}{E_s}$

$\Rightarrow \sigma_s = \dfrac{\sigma_c}{E_c} \times E_s$

$\Rightarrow \sigma_s = \sigma_c \times \dfrac{E_s}{E_c}$

$\Rightarrow \sigma_s = 3.5 \times 10^6 \times 18$

$\Rightarrow \sigma_s = 63000000 \ N/m^2$

$\Rightarrow \sigma_s = 63 \times 10^6 \ N/m^2$

$\sigma = \dfrac{P}{A}$

$\Rightarrow P = \sigma \times A$

Total load, $P = P_c + P_s$

$= (\sigma_c \times A_c) + (\sigma_s \times A_s)$

$= (3.5 \times 10^6 \times 49,500 \times 10^{-6}) + (63 \times 10^6 \times 500 \times 10^{-6})$

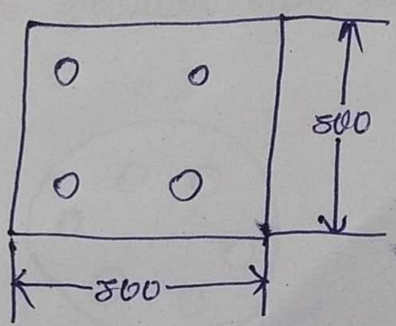
$= (173250 + 31500)$

$= 204750 \ N$

$= 204.75 \ KN$

Ex:- 8.16

A reinforced concrete colum 500mm X 500mm in section is reinforced with 4 steel bars of 25 mm diameter, one in each corner. the column is carrying a load of 1000 KN. Find the stresses in the concrete and ~~sect~~ steel bars.
Take E for steel = 210 GPa and E for concrete = 14 GPa.



Reinforced concrete

Area of column = $500 \times 500 = 2,50,000 \text{ mm}^2$

$= 250000 \times 10^{-6} \text{ m}^2$

No. of steel bar = 4

Diameter of steel bar (d) = 25mm =

Load on column (P) = $1000 \text{ KN} = 1000 \times 10^3 \text{ N}$

$E_s = 210 \text{ GPa} = 210 \times 10^9 \text{ Pa} = 210 \times 10^9 \text{ N/m}^2$

$E_c = 14 \text{ GPa} = 14 \times 10^9 \text{ Pa} = 14 \times 10^9 \text{ N/m}^2$

$A_s = 4 \times \frac{\pi}{4} \times (d^2) mm$

$= 4 \times \frac{\pi}{4} \times (25)^2$

$= \cancel{1963 mm^2} \; 1963 mm^2 = 1963 \times 10^{-6} m^2$

$A_c = 250000 - 1963 mm^2$

$= 248037 mm^2 = 248037 \times 10^{-6} m^2$

$\sigma_s = \frac{E_s}{E_c} \times \sigma_c$

$= \frac{210}{14} \times \sigma_c$

$= 15 \sigma_c$

$(P) : 1000 \times 10^3 = (\sigma_s . A_s) + (\sigma_c . A_c)$

$= (15 \sigma_c \times 1963 \times 10^{-6}) + (\sigma_c \times 248037 \times 10^{-6})$

$= \cancel{277482 \sigma_c} \; 0.277482$

$\sigma_c = \frac{1000 \times 10^3}{0.277482} = \cancel{3.6 N/mm^2 = 3.6 MPa} \; 3603837 N/m^2$

$\cancel{\sigma_s = 15 \sigma_c = 15 \times 3.6 = 54 MPa}$

$\sigma_s = 15 \sigma_c = 15 \times 3603837$

$= 54057555 \; N/m^2$

Thermal stresses and strains

Let l = Original length of the body,
t = Increase of temprature,
α = cofficient of linear expansion.

we know that the increase in length due to increase of temperature.

$$\delta l = l.\alpha.t$$

If the ends of the bar are fixed to rigid supports, so that its expansion is prevented, then compressive strain induced in the bar.

$$\varepsilon = \frac{\delta l}{l} = \frac{l\alpha.t}{l} = \alpha.t$$

$\therefore$ stress $\sigma = \varepsilon.E = \alpha.t.E$

Cor. If the supports yield by an amount equal to Δ, then the actual expansion that has taken place,

$$\delta l = l\alpha t - \Delta$$

strain $\varepsilon = \frac{\delta l}{l} = \frac{l\alpha t - \Delta}{l} = \left(\alpha t \frac{\Delta}{t}\right)$

$\therefore$ stress $\sigma = \varepsilon.E = \left(\alpha t - \frac{\Delta}{l}\right)E$

The value of α (i.e. coefficient of linear expansion) of materials in every day use are given below in table

S.No	materia	cofficient of linear expansion /°C (α)
1	steel	11.5×10^{-6} to 13×10^{-6}
2	wrought iron, cast iron	11×10^{-6} to 12×10^{-6}
3	Aluminium	23×10^{-6} to 24×10^{-6}
4	copper, Brass, Bronze	17×10^{-6} to 18×10^{-6}

Ex:- 5.1

A luminium alloy bar, fixed at its both ends is heated through 20K. Find the stress developed in the bar. Take modulus of elasticity, and coefficient of linear expansion for the bar material as 80GPa & $24\times10^{-6}/K$ respectively.

Aluminium alloy bar
fixed at both side

$t = 20K$

$E = 80\ GPa$

$= 80\times10^{3}\ N/m^2$

$\alpha = 24\times10^{-6}/K$

$\sigma = \alpha . t . E$

$= 24\times10^{-6}\times20\times80\times10^{3}$

$= 38.4\ N/mm^2$

$= 38.400000\ N/m^2$

Ex:-5.2

A brass rod 2m long is fixed at both its ends. If the thermal stress is not to exceed 76.5 MPa, calculate the temperature through which the rod should be heated. Take the values of α and E as $17\times10^{-6}/K$ & 90GPa respectively.

Brass rod
fixed at both side

$l = 2mm$

$\sigma_{max} = 76.5\ MPa = 76.5\ \cancel{\times000}\ N/mm^2$

$\alpha = 17\times10^{-6}/K$

$E = 90\ GPa = 90\times10^{3}\ N/mm^2$

t - Temperature through which the rod should be heated in K.

maximum stress in the rod (σ_{max})

$$76.5 = \alpha . t . E = (17 \times 10^{-6}) \times t \times (90 \times 10^{3}) = 1.53 t$$

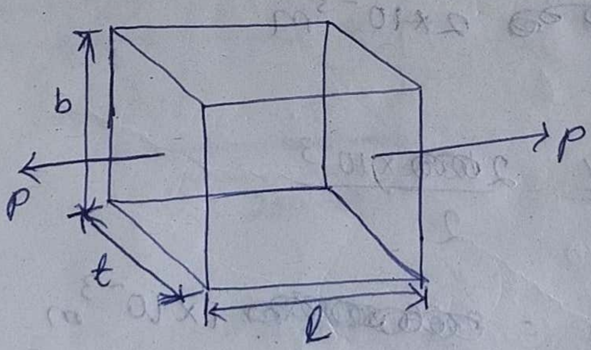
$$t = \frac{76.5}{1.53}$$

$$= 50 \text{ K}$$

Elastic Constant :-

Ex :- 6.1

A steel bar 2m long, 40mm wide and 20 mm thick is subjected to an axial pull of 160 KN in the direction of its length. Find the changes in length, width and thickness of the bar. Take E = 200 GPa and poisson's ratio = 0.3.



$l = 2m$

$b = 40 mm = 40 \times 10^{-3} m$

$t = 20 mm = 20 \times 10^{-3} m$

$P = 160 KN = 160 \times 10^{3} N$

$E = 200 GPa = 200 \times 10^{9} Pa = 200 \times 10^{9} N/m^2$

$\frac{1}{m} = 0.3$

force is applied along it's length

Cross sectional Area $(A) = b \times t$

$= 40 \times 20$

$= 800 \text{ mm}^2$

$= 800 \times 10^{-6} \text{ m}^2$

linear strain $= \frac{\delta l}{l}$

Lateral strain $= \frac{\delta b}{b}, \frac{\delta t}{t}$

$\delta l = ?$

$\delta b = ?$

$\delta t = ?$

$\delta l = \frac{PL}{AE}$

$= \frac{160 \times 10^3 \times 2}{800 \times 10^{-6} \times (200 \times 10^9)}$

$= 2 \times 10^{-3} \text{ m}$

linear strain

$\epsilon = \frac{\delta l}{l} = \frac{2 \times 10^{-3}}{2}$

$= 1 \times 10^{-3} \text{ m}$

lateral strain =

$\frac{1}{m} \times \epsilon = 0.3 \times 0.06$

$= 0.018$

$\frac{\delta b}{b}$ = lateral strain

$\Rightarrow \delta b = b \times$ lateral strain

$= 40 \times 10^{-3} \times 0.018$

$= 0.72 \times 10^{-3} \text{ m}$

poisson's ratio $= \frac{\text{Lateral strain}}{\text{linear strain}}$

$0.3 = \frac{\text{lateral strain}}{1 \times 10^{-3}}$

lateral strain $= 0.3 \times 1 \times$

$= 3 \times 10^{-4} \text{ m}$

$$\cancel{\delta t = t \times \text{Lateral strain}}$$
$$\cancel{= 20\times10^{-3} \times 0.018}$$
$$\cancel{= 0.36\times10^{-3}\,m}$$

Lateral strain $= \frac{\delta b}{b}$

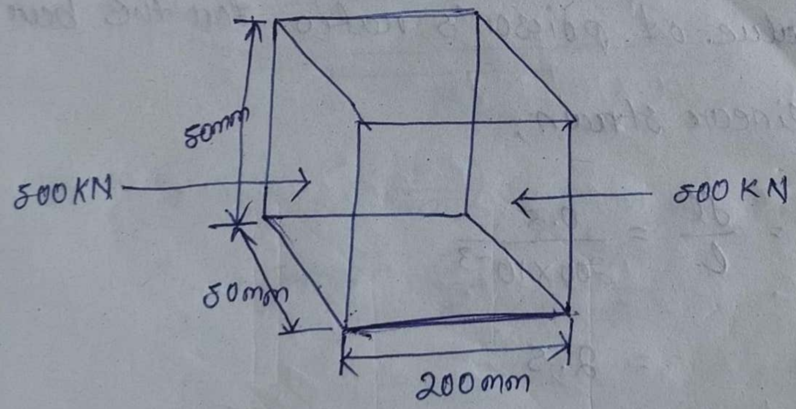
$$3\times10^{-4} = \frac{\delta b}{40\times10^{-3}}$$

$$\delta b = 3\times10^{-4} \times 40\times10^{-3}$$
$$= 1.2\times10^{-5}\,m$$

$$\frac{\delta t}{t} = 3\times10^{-4} \quad \frac{\delta t}{2\times10^{-3}}$$
$$\delta t = 3\times10^{-4}\times 20\times10^{-3}$$
$$= 6\times10^{-6}\,m$$

Exi- 6.2

A metal bar 50mm x 50mm in section is subjected to an axial compressive load of 500 KN. If the contraction of a 200mm gauge length was found to be 0.5mm and the increase in thickness 0.04 mm, find the values of young's modulus and poisson's ratio for the bar material.



$b = 50mm = 50\times10^{-3}\,m$

$t = 50\,mm = 50\times10^{-3}\,m$

$P = 500KN = 500\times10^{3}\,N$

$l = 200\,mm = 200\times10^{-3}\,m$

$\delta l = 0.5\,mm = 0.5\times10^{-3}\,m$

$\delta t = 0.04\,mm = 0.04\times10^{-3}\,m$

$A = 50\,mm \times 50\,mm = 2500\,mm$

$= 2500\times10^{-3}\,m$

Value of young's modulus for the bar material

E = value of young's modulus for the bar meterial.

we know that contraction of the bar (δl).

$$0.5\times10^{-3} = \frac{Pl}{AE} = \frac{500\times10^{3}\times200\times10^{-3}}{2500\times10^{-3}\times E}$$

$$= \frac{40\times10^{3}}{E}$$

$$E = \frac{40\times10^{3}}{0.5} = 80\times10^{3}\ N/mm^{2}$$

$$= 80\times10^{9}\ N/m^{2}$$

value of poisson's ratio for the bar material.

$\frac{1}{m}$ = value of poisson's ratio for the bar mater

we know that linear strain,

$$\varepsilon = \frac{\delta l}{l} = \frac{0.5}{200\times10^{-3}}$$

$$= 2.5$$

lateral strain

$$\frac{1}{m}\times \text{linear strain}$$

$$= \frac{1}{m}\times2.5$$

we also know that increase in thickness (δt),

$$\delta t = t\times \text{lateral strain}$$

$$0.04\times10^{-3} = 50\times10^{-3}\times\frac{1}{m}\times2.5 = \frac{0.125}{m}$$

$$\frac{1}{m} = \frac{0.04\times10^{-3}}{0.125} = 0.00032$$

$$= 0.32\times10^{-3}$$

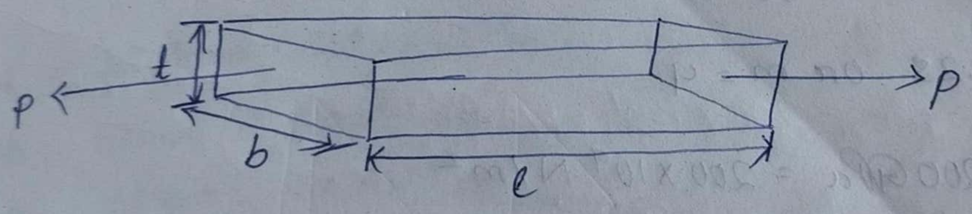
Volumetric strain :-

It may be defined as the change in volume, to the original volume

Mathematically,

$$e_v = \frac{\text{Change in volume}}{\text{Original volume}}$$

volumetric strain of a rectangular body subjected to an axial force.



$$e_v = e\left(1 - \frac{2}{m}\right)$$

$$e = \text{strain} = \frac{\sigma}{E} = \frac{(P/A)}{E}$$

$$= \frac{P}{AE}$$

$$= \frac{P}{b.t.E}$$

$$\frac{1}{m} = \text{poisson's Ratio}$$

Ex :- 6.3

A steel bar 2m long, 20mm wide and 15mm thick is subjected to a tensile load of 30 KN. Find the increase in volume, if poisson's ratio is 0.25 and young's modulus is 200 GPa.

$l = 2m$

$b = 20mm = 20 \times 10^{-3} m$

$t = 15mm = 15 \times 10^{-3} m$

$P = 30 KN = 30 \times 10^{3} N$

$\frac{1}{m} = 0.25$ or $m = 4$

$E = 200 GPa = 200 \times 10^{9} N/m^2$

δv = Increase in volume of the bar, we know that original volume of the bar.

$$V = l.b.t = (2 \times 20 \times 10^{-3} \times 15 \times 10^{-3})$$

$$= 6 \times 10^{-4}$$

$$\frac{\delta v}{V} = \frac{P}{btE}\left(1 - \frac{2}{m}\right)$$

$$= \frac{30 \times 10^{3}}{20 \times 10^{-3} \times 15 \times 10^{-3} \times 200 \times 10^{9}}$$

$$= 2.5 \times 10^{-4}$$

$\delta v = 2.5 \times 10^{-4} \times v$

$= 2.5 \times 10^{-4} \times 6 \times 10^{-4}$

$= 1.5 \times 10^{-7} m^3$

Ex:- 6.4

A copper bar 250mm ~~long~~ long and 50mm x 50mm in cross-section is subjected to an axial pull in the direction of its length. If the increase in volume of the bar is $37.5 mm^3$, find the magnitude of the pull. Take m = 4 and E = 100GPa.

$l = 250mm = 250 \times 10^{-3} m$

$b = 50mm = 50 \times 10^{-3} m$

$t = 50mm = 50 \times 10^{-3} m$

$\delta v = 37.5 = 37.5 \times 10^{-4} m^3$

$m = 4$

$E = 100 GPa = 100 \times 10^{9} N/m^2$

P = magnitude of the pull in KN.

we know that original volume of the copper bar,

$V = l.b.t = (250 \times 10^{-3} \times 50 \times 10^{-3} \times 50 \times 10^{-3})$

$= 6.25 \times 10^{-4} m^3$

$$\frac{\delta v}{V} = \frac{P}{btE}\left(1 - \frac{2}{m}\right) = \frac{P}{50 \times 10^{-3} \times 50 \times 10^{-3} \times 100 \times 10^{9}}\left(1 - \frac{2}{4}\right)$$

$$\frac{37.5 \times 10^{-4}}{6.25 \times 10^{-4}} = \frac{P}{2 \times 10^{-9}}$$

$$\Rightarrow P = \frac{37.5 \times 10^{-4} \times 2 \times 10^{-9}}{6.25 \times 10^{-4}} = 0.0012$$

$= 1.2 \times 10^{-3} N$

Ex :- 6.5

A steel bar 50mm x 50mm in cross-sectinon is 1.2m long. It is subjected to an axial pull of 200KN. ~~wsts~~ what are the changes in lenth, width and volume of the bar, If the volume of poisson's ratio is 0.3? Take E as 200 GPa.

$A = 50 \times 50 = 2500\ mm = 2500 \times 10^{-3} m$

$b = 50mm = 50 \times 10^{-3} m$

$t = 50mm = 50 \times 10^{-3} m$

$l = 1.2m$

$P = 200\ KN = 200 \times 10^{3} N$

$\frac{1}{m} = 0.3$

$E = 200\ GPa = 200 \times 10^{9}\ N/m^2$

Change in length

we know that change in lenth,

$$\delta l = \frac{Pl}{AE} = \frac{200 \times 10^{3} \times 1.2}{2500 \times 10^{-3} \times 200 \times 10^{9}}$$

$$= 4.8 \times 10^{-7} m^2$$

Change in width

we know that linear strain,

$$\varepsilon = \frac{\delta l}{l} = \frac{4.8 \times 10^{-7}}{1.2} = 4 \times 10^{-7} m^2$$

lateral strain

$$\frac{1}{m} \times \varepsilon = 0.3 \times 4 \times 10^{-7} = 1.2 \times 10^{-7}$$

∴ changein width

$$\delta b = b \times \text{lateral strain}$$

$\delta b = 50 \times 10^{-3} \times 1.2 \times 10^{-7}$

$= 6 \times 10^{-9}\ m^2$

Change in volume

we also know that volume of the bar.

$V = l.b.t = (1.2 \times 50 \times 10^{-3} \times 50 \times 10^{-3})$

$= 3 \times 10^{-3}$

$$\frac{\delta v}{V} = \frac{P}{btE}\left(1 - \frac{2}{m}\right) = \frac{200 \times 10^3}{50 \times 10^{-3} \times 50 \times 10^{-3} \times 200 \times 10^9}(1 - 2 \times 0.3)$$

$= 1.6 \times 10^{-4}$

$\delta v = 1.6 \times 10^{-4} \times v$

$= 1.6 \times 10^{-4} \times 3 \times 10^{-3}$

$= 4.8 \times 10^{-7}\ m^3$

Bulk Modulus:-

→ It may be defined as the ratio of direct stress to the volumetric strain.

→ It is denoted by K

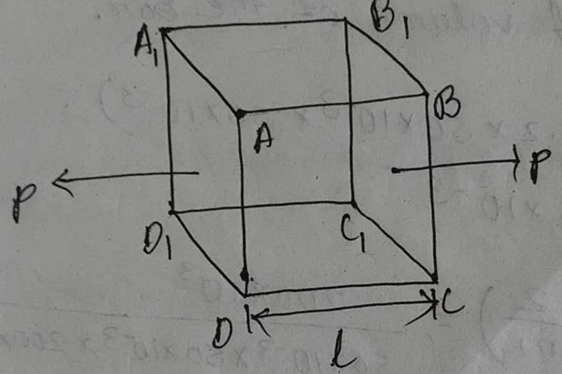
→ Mathematically

$$K = \frac{\text{Direct stress}}{\text{volumetric strain}} = \frac{\sigma}{e_v}$$

σ = Direct stress $= \frac{P}{A}$

e_v = volumetric stress $= \frac{\delta v}{V}$

Relation between bulk modulus and young's modulus :-



Consider a cube ABCD $A_1B_1C_1D_1$ as shown in fig. Let the cube be subjected to three mutually perpendi[cular] tensile stresses of equal intensity.

Let σ = Stress on the faces,

l = Lenth of the cube, and

E = young's modulus for the material of the block.

Now consider the deformation of one side of cube under the action of the three mutually perpendicular Stress. we know that side will suffer the following strains due to the pair of stresses.

1. Tensile Strain equal to $\frac{\sigma}{E}$ due to stress on the faces BB_1 CC_1 and AA_1 DD_1

2. compressive lateral strain equal to due to stress on faces AA_1 BB_1 and DD_1 CC_1

3. compressive lateral strain equal to $\frac{1}{m} \times \frac{\sigma}{E}$ due to stresses on faces ABCD and $A_1B_1C_1D_1$.

Therefore net tensile strain, which the side AB will suffer, due to these stresses,

$$\frac{\delta l}{l} = \frac{\sigma}{E} - \left(\frac{1}{m} \times \frac{\sigma}{m}\right) - \left(\frac{1}{m} \times \frac{\sigma}{E}\right) = \frac{\sigma}{E}\left(1 - \frac{2}{m}\right)$$

We know that the original volume of the cub,

$$V = l^3$$

Differentiating the above equation with respect to l,

$$\frac{\delta V}{\delta l} = 3l^2$$

or $\delta V = 3l^2 . \delta l = \cancel{3l^3 \times \frac{\delta l}{l}}$

$$= 3l^3 \times \frac{\delta l}{l}$$

Substituting the value of $\frac{\delta l}{l}$ from equation (1)

$$\delta V = 3l^3 \times \frac{\sigma}{E}\left(1 - \frac{2}{m}\right)$$

$$\frac{\delta V}{V} = \frac{3l^3}{l^3} \times \frac{\sigma}{E}\left(1 - \frac{2}{m}\right)$$

$$= \frac{3\sigma}{E}\left(1 - \frac{2}{m}\right)$$

$$\frac{\sigma}{\frac{\delta V}{V}} = \frac{E}{3} \times \frac{1}{\left(1 - \frac{2}{m}\right)} = \frac{E}{3} \times \frac{1}{\left(\frac{m-2}{m}\right)}$$

$$\boxed{K = \frac{mE}{3(m-2)}}$$

Ex:- 6.9

If the values of modulus of elasticity and poisson's ratio for an alloy body is 150 GPa and 0.25 respectively, determine the value of bulk modulus for the alloy.

$E = 150\ GPa = 150 \times 10^9\ N/m^2$

$\frac{1}{m} = 0.25$

$m = 4$

we know that value of the bulk modulus for the alloy,

$$K = \frac{mE}{3(m-2)}$$

$$= \frac{4 \times 150 \times 10^9}{3(4-2)}$$

$$= 1. \times 10^{11}\ N/m^2$$

Ex:- 6.10

For a given material, young's modulus is 120 GPa and modulus of rigidity is 40 GPa. Find the bulk modulus and lateral contraction of a round bar of 50mm diameter and 2.5m long, when stretched 2.5mm. Take poisson's ratio as 0.25.

$E = 120\ GPa = 120 \times 10^9\ N/m^2$

$C = 40\ GPa = 40 \times 10^9\ N/m^2$

$d = 50\ mm = 50 \times 10^{-3} m^2$

$l = 2.5 m$

$\delta l = 2.5 \text{ mm} = 2.5 \times 10^{-3} m2$

$\frac{1}{m} = 0.25$

$m = 4$

Bulk modulus of the bare

we know that bulk modulus of the bar,

$$K = \frac{mE}{3(m-2)}$$

$$= \frac{4 \times 120 \times 10^{9}}{3(4-2)}$$

$$= 8 \times 10^{10} \text{ N/m}^2$$

Lateral contraction of the bare

contraction of the bar

δd = lateral contraction

we know that linear strain

$$\varepsilon = \frac{\delta l}{l} = \frac{2.5 \times 10^{-3}}{2.5} = 1 \times 10^{-3}$$

and lateral strain

$$\frac{\delta d}{d} = \frac{1}{m} \times \varepsilon = 0.25 \times 1 \times 10^{-3}$$

$$= 2.5 \times 10^{-4}$$

$$\delta d = d \times 2.5 \times 10^{-4}$$

$$= 50 \times 10^{-3} \times 2.5 \times 10^{-4}$$

$$= 1.25 \times 10^{-5} \text{ m}^2$$

Shear modulus or modulus of Rigidity :-

It has been experimentally found that within the elastic limit, the shear stress is proportional to the shear strain.

Mathematically,

$$\tau \propto \phi$$

$$\tau = C \times \phi$$

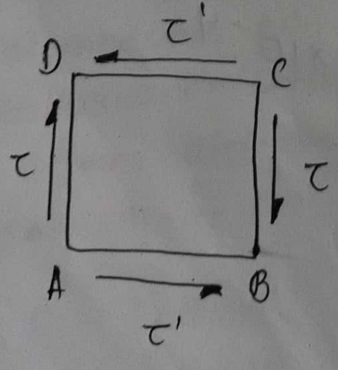
$$\frac{\tau}{\phi} = C \text{ (or } G \text{ or } N)$$

τ = shear stress

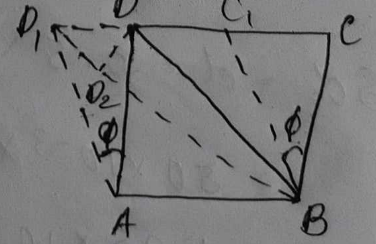
ϕ = shear strain

C = A constant, known as shear modulus or modulus of rigidity. It is also denoted by G or N.

Relation between modulus of Elasticity and Modulus of Rigidity :-



(a. Before distortion)



(b. After distorition)

Consider a cube of length l subjected to a shear stress of τ as shown in fig. (a) A little consideration will show that due to these stresses the cube is subjected to some distortion, such that the diagonal BD will be elongated and the diagonal AC will be shorted. Let this shear stress & cause shear strain ϕ as shown in fig. (b) we see that the diagonal BD is now distorted to BD_1

$$\text{strain of } BD = \frac{BD_1 - BD}{BD} \qquad \left(\because \text{strain} = \frac{\delta l}{l}\right)$$

$$= \frac{D_1D_2}{BD} = \frac{DD_1 \cos 45^\circ}{AD\sqrt{2}}$$

$$= \frac{DD_1}{2AD}$$

$$= \frac{\phi}{2}$$

Thus we see that the linear strain of the diagonal BD is half of the shear strain and is tensile in nature. Similarly it can be proved that the linear strain of the diagonal AC is also equal to half of the shear strain, but is compressive in nature. Now this linear strain of the diagonal BD.

$$\frac{\phi}{2} = \frac{\tau}{2C} \quad \text{———} (i)$$

τ = shear stress

C = modulus of rigidity

Let us now consider this shear stress τ acting on the sides AB, CD, CB and AD, we know that the effect of this stress is to cause tensile stress on the diagonal BD and compressive stress on the diagonal AC. Therefore tensile strain on the diagonal BD due to tensile stress on the diagonal BD

$$\frac{\tau}{E} \quad \text{———} (ii)$$

and the tensile strain on the diagonal BD due to compressive stress on the diagonal AC

$$\frac{1}{m} \times \frac{\tau}{E} \quad \text{———} (iii)$$

The combined effect of the above two stresses on the diagonal BD

$$\frac{\tau}{E} + \frac{1}{m} \times \frac{\tau}{E}$$

$$= \frac{\tau}{E}\left(1 + \frac{1}{m}\right)$$

$$= \frac{\tau}{E}\left(\frac{m+1}{m}\right) \quad \text{——— (iv)}$$

Equating equations (i) and (iv),

$$\frac{\tau}{2C} = \frac{\tau}{E}\left(\frac{m+1}{m}\right)$$

or

$$C = \frac{mE}{2(m+1)}$$

Ex:- 6.11

An alloy specimen has a modulus of elasticity of 120 GPa and modulus of rigidity of 45 GPa. Determine the poisson's ratio of the material.

$E = 120$ GPa $= 120 \times 10^9$ N/m²

$C = 45$ GPa $= 45 \times 10^9$ N/m²

$\frac{1}{m}$ = poisson's ratio of the materials

We know that the modulus of rigidity (C),

$$C = \frac{mE}{2(m+1)}$$

$$45 = \frac{m \times 120 \times 10^9}{2(m \times 1)}$$

$$= \frac{120 \times 10^9 m}{2m \times 2}$$

Ex: - 6.12

In an experiment, a bar of 30 mm diameter is subjected to a pull of 60 KN. The measured extension on gauge length of 200 mm is 0.09 mm and the change in diameter is 0.0039 mm. Calculate the poisson's ratio and the values of the three moduli.

$d = 30 mm = 30 \times 10^{-3} m$

$P = 60 KN = 60 \times 10^{3} N$

$l = 200 mm = 200 \times 10^{-3} m$

$\delta l = 0.09 mm = 0.09 \times 10^{-3} m$

$\delta d = 0.0039 mm = 0.0039 \times 10^{-3} m$

poisson's ratio

we know that linear strain,

$$\varepsilon = \frac{\delta l}{l} = \frac{0.09 \times 10^{-3}}{200 \times 10^{-3}}$$

$$= 4.5 \times 10^{-4}$$

lateral strain

$$\frac{\delta d}{d} = \frac{0.0039 \times 10^{-3}}{30 \times 10^{-3}}$$

$$= 1.3 \times 10^{-4}$$

we know that poisson ratio

$$\frac{1}{m} = \frac{\text{lateral strain}}{\text{linear strain}} = \frac{1.3 \times 10^{-4}}{4.5 \times 10^{-4}}$$

$$= 0.289$$

value of three moduli

Let E = value of young's modulus

~~and extension of the bar (δl).~~

we know that area of the bar.

$$A = \frac{\pi}{4} \times (d)^2$$

$$= \frac{\pi}{4} (30 \times 10^{-3})^2$$

$$= 7.068583471 \times 10^{-4} \ m^2$$

and extension of the bar (δl).

$$\delta l = \frac{P.l}{AE}$$

$$0.09 \times 10^{-3} = \frac{60 \times 10^3 \times 200 \times 10^{-3}}{7.068583471 \times 10^{-4} \times E}$$

$$= \frac{16,976,527.26}{E}$$

$$\Rightarrow E = \frac{16976527.26}{0.09 \times 10^{-3}}$$

$$= 1.88614114 \times 10^{10} \ N/m^2$$

We know from the value of poisson's ratio that

$$m = \frac{1}{0.289} = 3.46$$

and value of modulus of rigidity

$$C = \frac{mE}{2(m+1)} = \frac{3.46 \times 1.8861414 \times 10^{10}}{2(3.46+1)} \ N/m^2$$

$$= 7,316,198,704$$

$$= 7.316 \times 10^{9} \ N/m^2$$

we also know that the value of bulk modulus,

$$K = \frac{m.E}{3(m-2)}$$

$$= \frac{3.46 \times 1.8861414 \times 10^{10}}{3(3.46-2)}$$

$$= 1.489 \times 10^{9} \ N/m^2$$

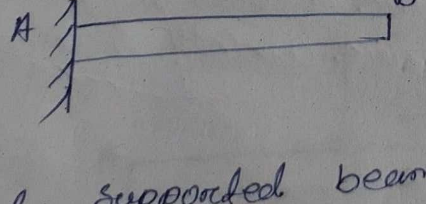
Bending moment and Shear force :-

Types of Beams and Loading :-

Types of Beams :-

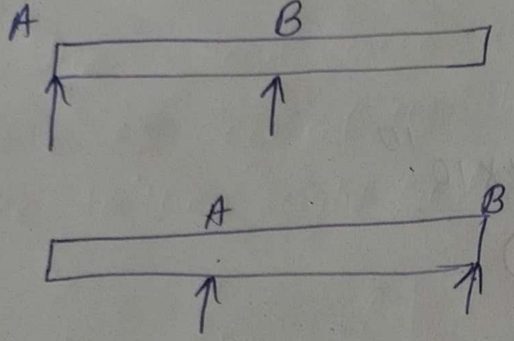
The types of beams are classified as under :

1) Cantilever beam

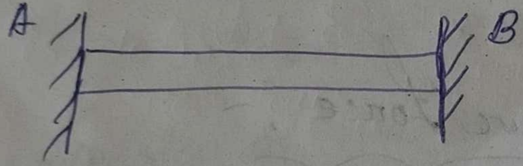


2) Simply supported beam

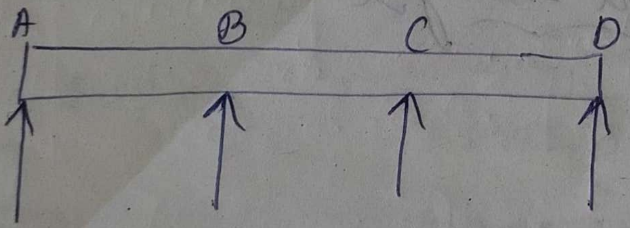
3) Overhanging beam



4) Rigidly fixed or built-in beam



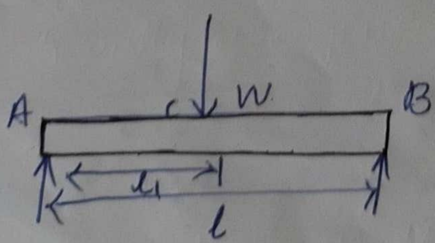
5) Continuous beam



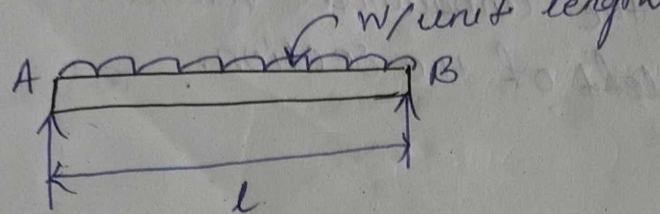
Types of loading :-

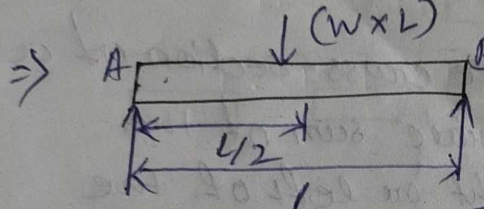
A beam may be subjected to either or in combinati of the following types of loads:

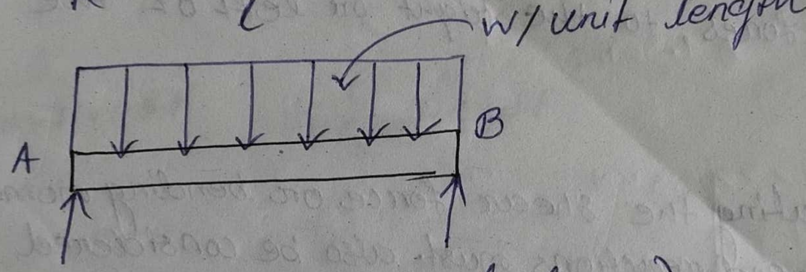
1) Point load :-



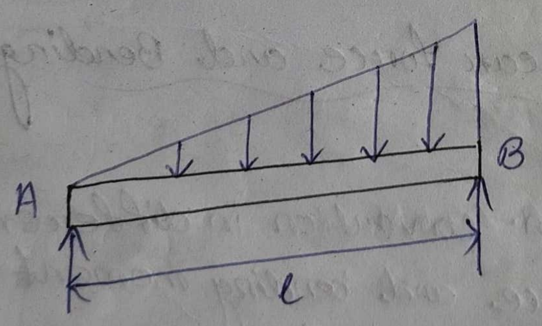
2) Uniformly Distributed load (UDL)

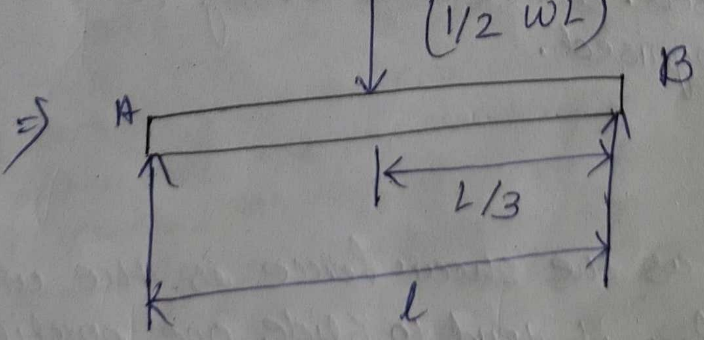






3) Uniformly varying load (UVL)





Shear Force :-

The shear force at the cross-section of a beam, is defined as the unbalanced vertical force to the right or left of the section.

Bending moment :-

The bending moment at the cross-section of a beam, is defined as the algebraic sum of the moments of the forces, to the right or left of the section.

Note

while calculating the shear force or bending moment at a section, the end reactions must also be considered alongwith other external loads.

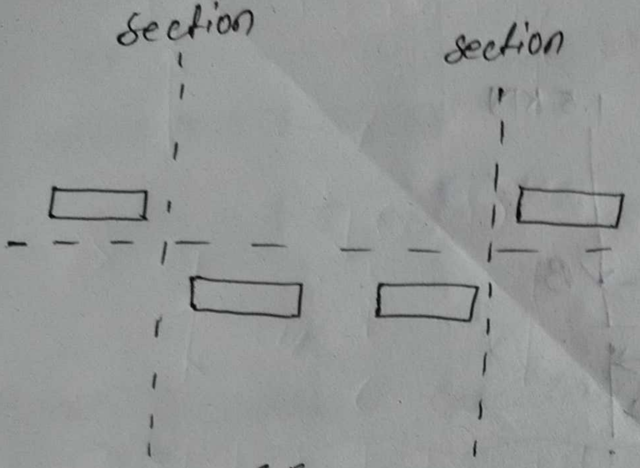
Sign Conventions for Shear force and Bending moment :-

We find different sign convention in different books, regarding shear force and bending moment at a section. But in this book the following sign conventions will be used, which are widely followed and internationally recognised.

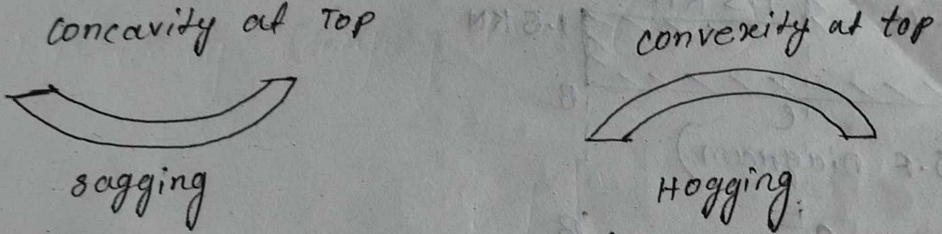
1) Shear force

We know that as the shear force is the unbalance vertical force, therefore it tend to slide one portion of the beam, upwords or downwards with respect to the other. we take shear force at a section as positive, when the left hand portion tends to slide upwards or the right hand portion tends to slide downwards show in fig.

Similarly, we take shear force at a section as negative, when the left hand portion tends to slide downwards or the right hand portion tends to slide upwards as now in fig.



a) positive S.F b) Negative S.F



c) positive B.M d) Negative B.M

2) Bending Moment : We take the bending moment at a section as positive, if it tends to bends the beam at that point to a curvature having concavity at the top, as shown in fig.

on the other hand, we take the bending moment at a section as negative, if it tends to bend the beam that point to a curvature having convexity at the top, as shown in fig

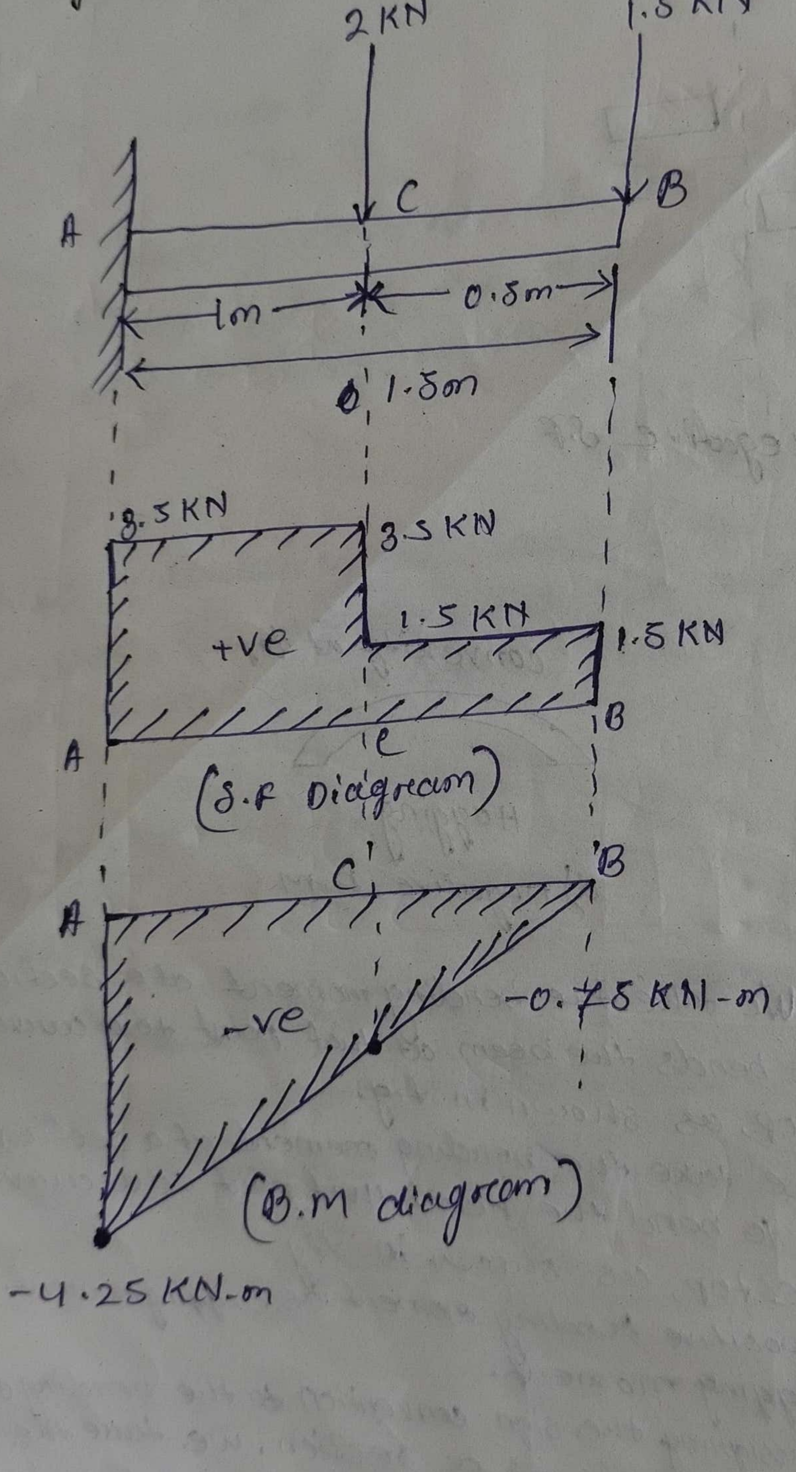
we often call the positive bending moment as sagging moment & negative bending moment as hogging moment.

Another way of assigning the sign convention to the bending moment is by the direction in which it acts at a section. we take the bending moment at a section as positive, when it is acting in clockwise direction to the left or in anticlockwise direction to the right.

on the other land, we take the bending moment at a section as negative, when it is acting in anticlockwise direction to the left or in clockwise direction to the right.

Ex :- 13.1

Draw shear force and bending moment diagrams for a cantilever beam of span 1.5 m carrying point loads as shown in fig.



Shear force calculation

at B = +1.5 KN

up to C = + 1.5 KN

at C = 1.5 + 2 = 3.5 KN

up to A = 3.5 KN

at = 3.5 KN

Bending moment calculation

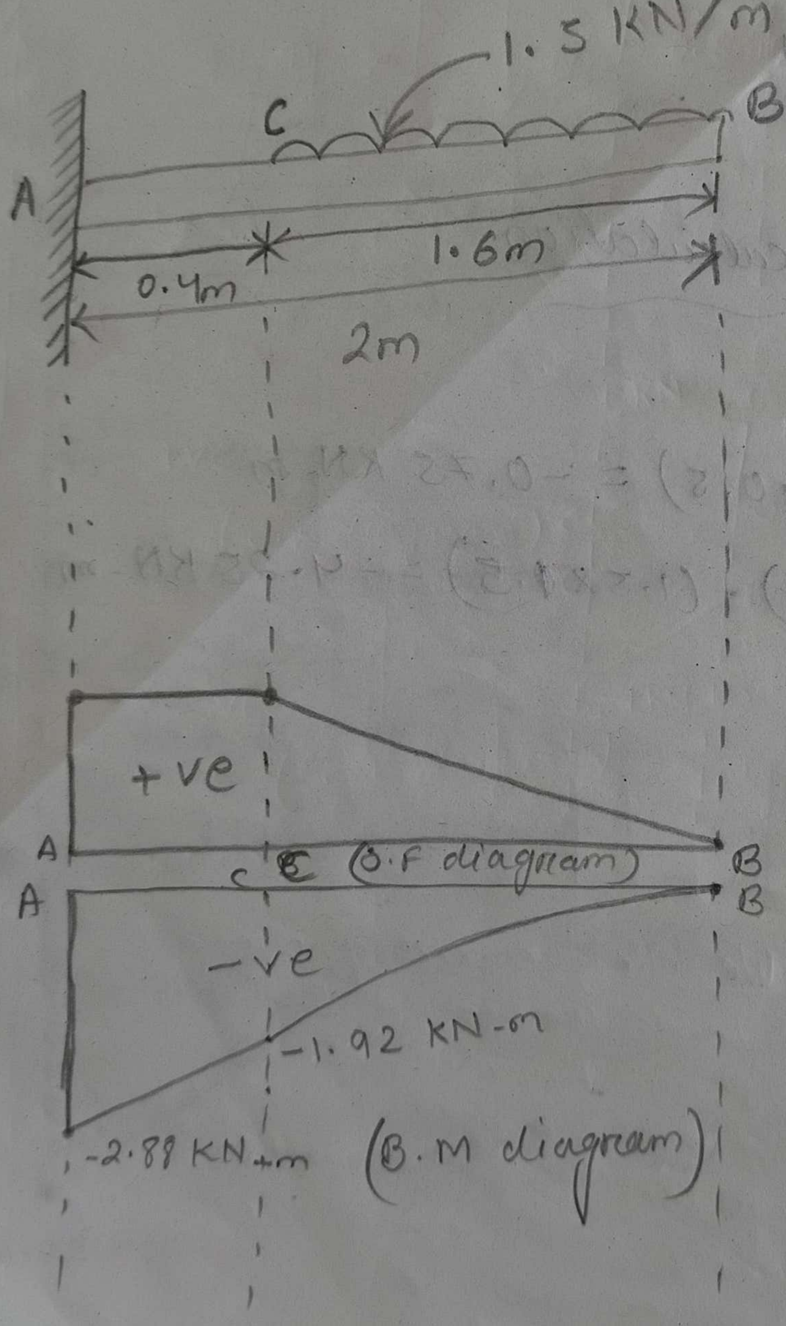
at B = 0

at C = $-(1.5 \times 0.5) = -0.75$ KN-m

at A = $-(2 \times 1) - (1.5 \times 1.5) = -4.25$ KN-m

Ex:-
13.2

A cantilever beam AB, 2m long carries a uniform distributed load of 1.5 KN/m over a length of 1.6m from the free end. Draw shear force and bending moment diagrams for the beam.



S.F. Calculation:-

at B = 0

up to C = $0 + (1.5 \times 1.6) = 2.4$ KN

at C = 2.4 KN

up to A = 2.4 KN

at A = 2.4 KN

B.M. calculation

~~at B = 0~~

~~at C = $-(1.5 \times 1.6) = -2.4$ KN~~

~~at A = $(1.5 \times 1.6) - (1.5 \times 2) =$~~

~~at B = 0~~

~~at C = $-(1.5 \times 1.6) = -2.4$~~

~~at A = $-(1.5 \times 2) - 0.4 = -1.2$~~

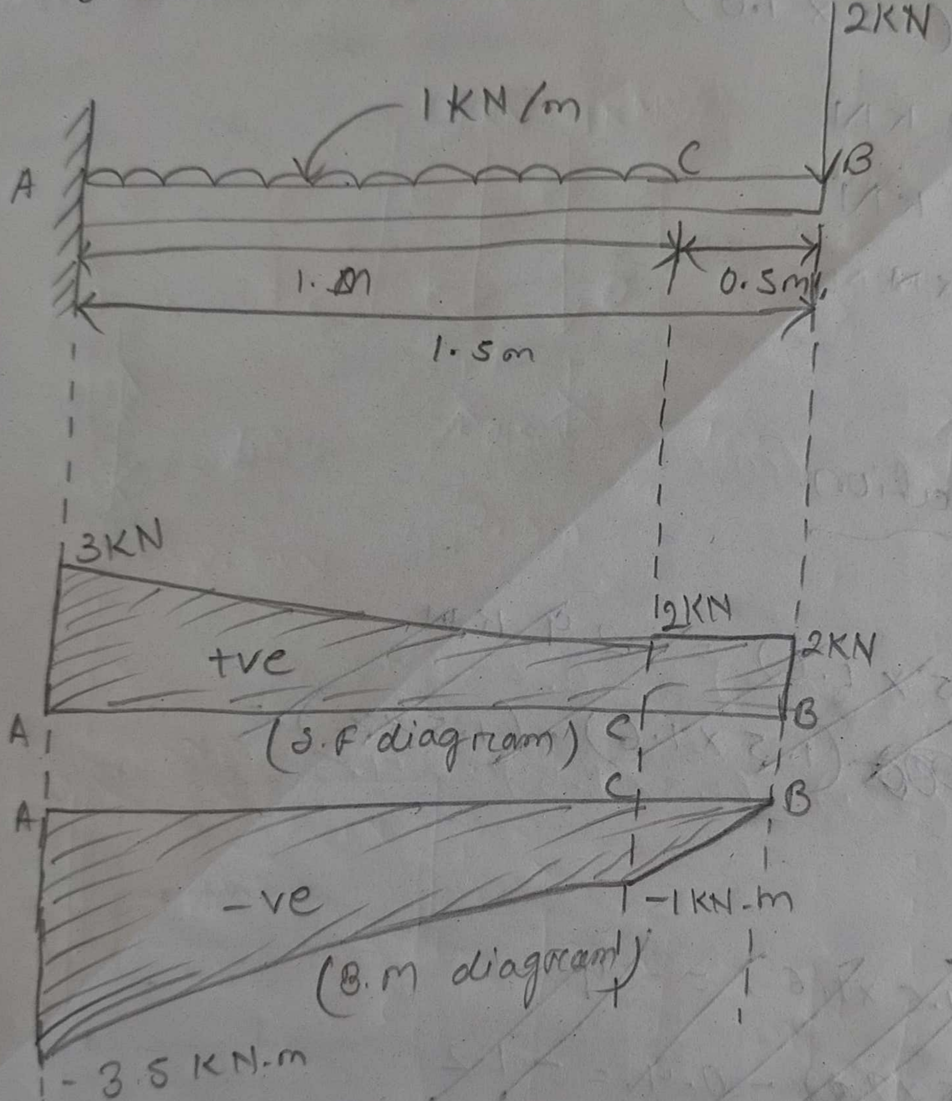
at B = 0

at C = $-\frac{wa^2}{2} = \frac{1.5 \times (1.6)^2}{2} = -1.92$ KN-m

at A = $-\left\{(1.5 \times 1.6)\left(0.4 + \frac{1.6}{2}\right)\right\} = -2.88$ KN-m

Q.13.3

A cantilever beam of 1.5m span is loaded as shown in fig. Draw the shear force and bending moment diagram.



S.F calculation

at B = +2 KN

up to C = 2 KN
at C = 2 KN
upto A = 2+(1×1) = 3 KN

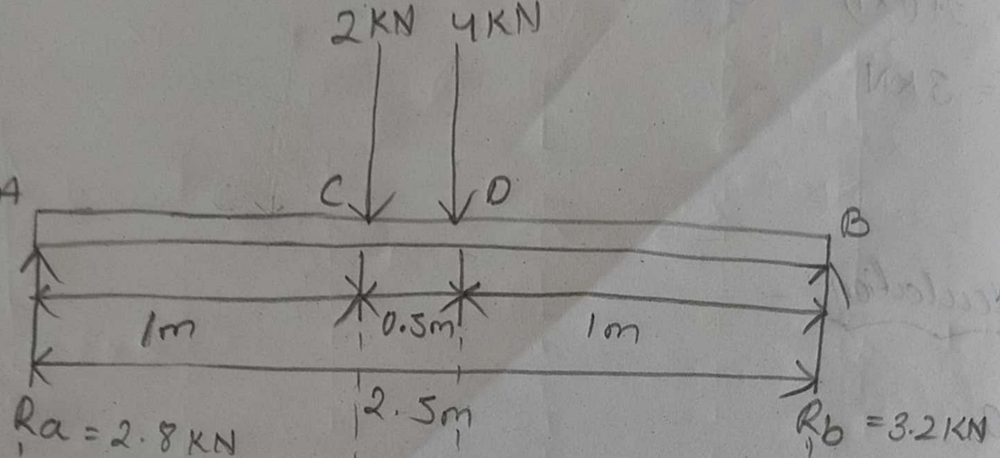
~~upto~~ at A = 3 KN

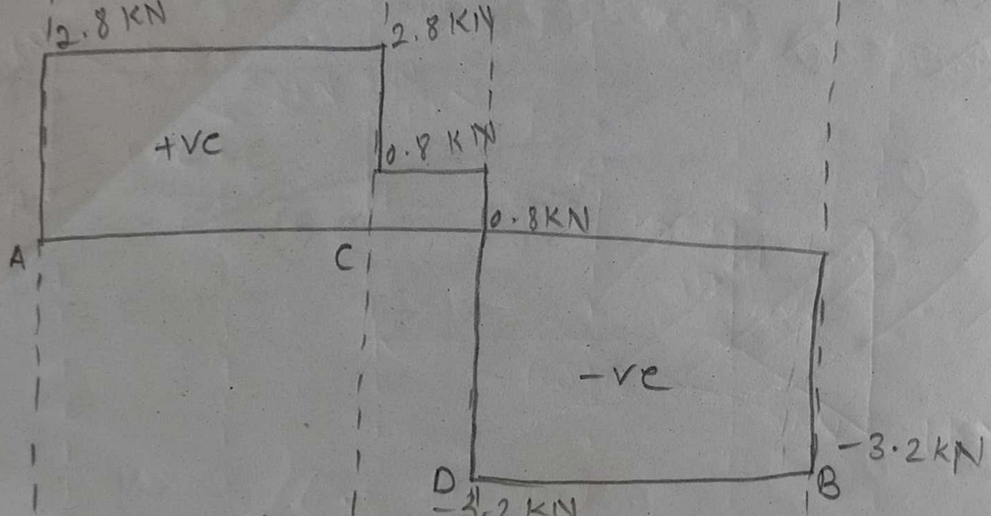
B.M calculation :-

at B = 0
at C = −(2×0.5) = −1 KN-m
at A = −(2×1.5) −(1×1×0.5) = −3.5 KN-m

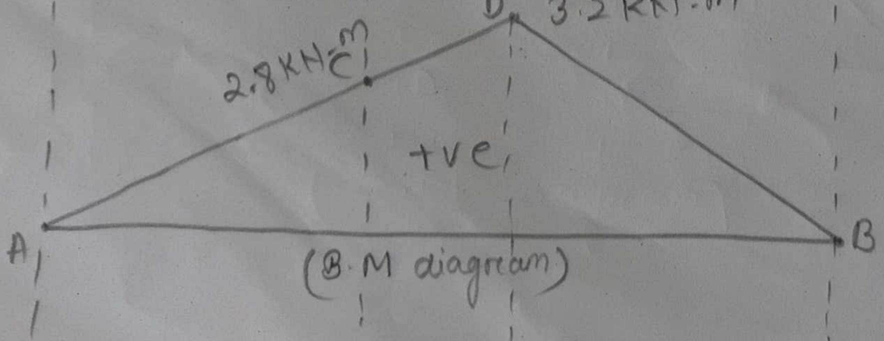
Ex - 13.6

A simply supported beam AB of span 2.5m is carrying the point loads as shown in fig. Draw the shear force and bending moment diagrams for the beam.





(S.F diagram)



(B.M diagram)

Taking moment about 'A' point

$\Sigma M_A = 0$

$\Rightarrow \cancelto{0}{(R_a \times 0)} + (R_b \times 2.5) - (2 \times 1) - (4 \times 1.5) = 0$

$\Rightarrow (R_b \times 2.5) = (2 \times 1) + (4 \times 1.5)$

$\Rightarrow R_b \times 2.5 = 8$

$\Rightarrow R_b = \frac{8}{2.5}$

$\Rightarrow R_b = 3.2$ KN

$\Sigma F_y = 0$

$\Rightarrow +R_a + R_b - 2 - 4 = 0$

$\Rightarrow R_a = 2 + 4 - R_b$

$\Rightarrow R_a = 6 - 3.2$

$\Rightarrow R_a = 2.8$ KN

S.F calculation :-

at B = -3.2 KN
up to D = -3.2 KN
at D = $-3.2 + 4 = 0.8$ KN
up to C = 0.8 KN
at C = $0.8 + 2 = 2.8$ KN
up to A = 2.8 KN
at A = 2.8 KN

The shear force changes it ~~sligh~~ sign from -ve to +ve or vice versa at 'D' point Hence the bending moment will be maxium at this point.

B.M calculation :-

at B = $R_b \times 0 = 0$

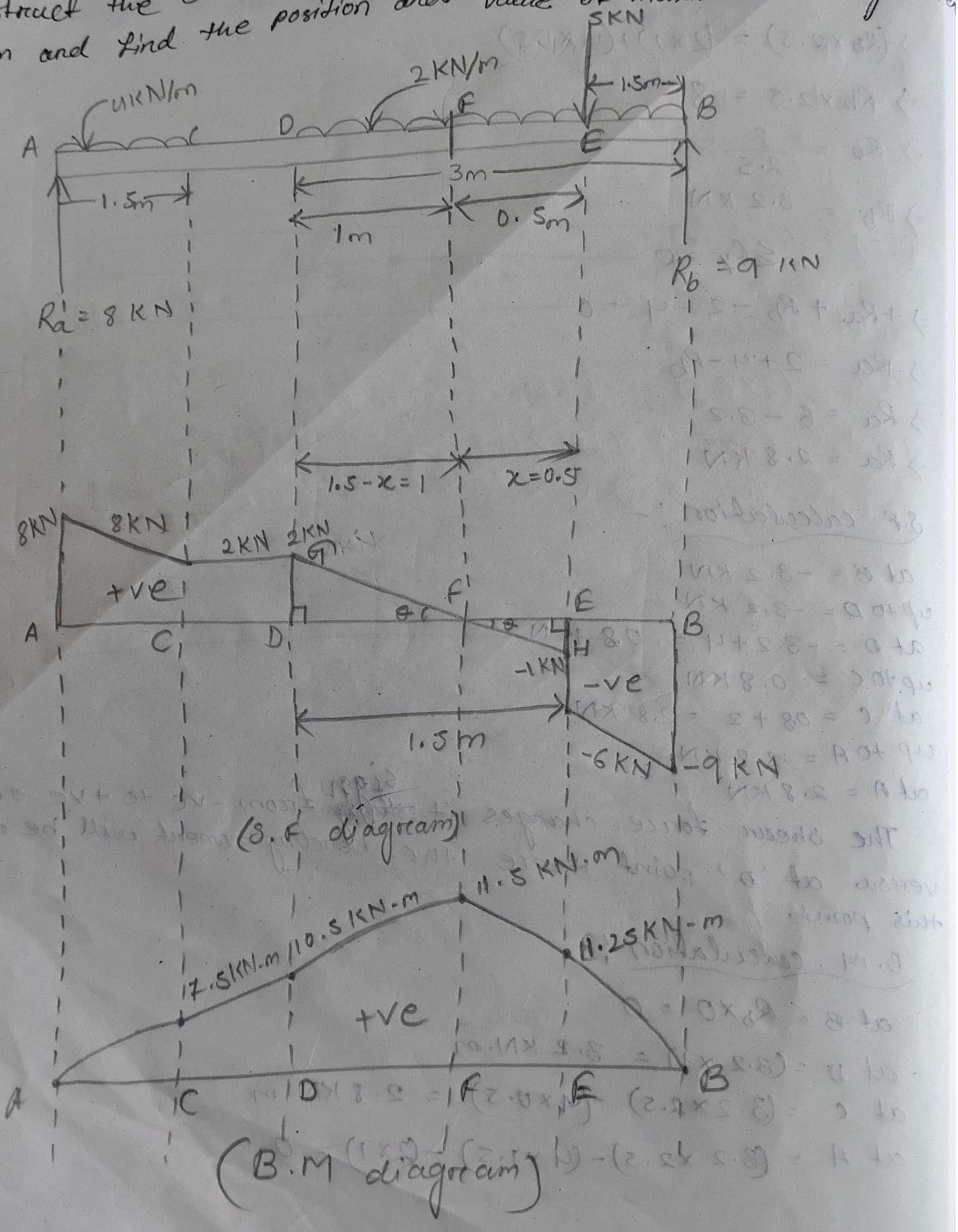
at D = $(3.2 \times 1) = 3.2$ KN.m

at C = $(3.2 \times 2.5) - (4 \times 0.5) = 2.8$ KN-m

at A = $(3.2 \times 2.5) - (4 \times 1.5) - (2 \times 1) = 0$

Ex :- 18.10

A simply supported beam AB, 6m long is loaded as shown. Construct the shear force and bending moment diagram for the beam and find the position and value of maximum bending moment.



Taking moment about 'A' point,

$$\sum M_A = 0$$

$\Rightarrow \cancel{(R_a \times 0)}^{0} + (R_b \times 6) - (5 \times 4.5) - (2 \times 3 \times 4.5) - (4 \times 1.5 \times \frac{1.5}{2}) = 0$

$\Rightarrow R_b \times 6 = (5 \times 4.5) + (2 \times 3 \times 4.5) + (4 \times 1.5 \times \frac{1.5}{2})$

$\Rightarrow R_b \times 6 = 54$

$\Rightarrow R_b = \frac{54}{6}$

$\Rightarrow R_b = 9$ KN

$$\sum F_y = 0$$

$\Rightarrow +R_a + R_b - 5 - (2 \times 3) - (4 \times 1.5) = 0$

$\Rightarrow R_a = 5 + (2 \times 3) + (4 \times 1.5) - R_b$

$\Rightarrow R_a = 5 + (2 \times 3) + (4 \times 1.5) - 9$

$\Rightarrow R_a = 8$ KN

S.F calculation :-

at B = -9 KN

up to E = $-9 + (2 \times 1.5) = -6$ KN

at E = $-6 + 5 = -1$ KN

up to D = $-1 + (2 \times 1.5) = 2$ KN

at D = 2 KN

up to C = 2 KN

at C = 2 KN

up to A = $2 + (4 \times 1.5) = 8$ KN

at A = 8 KN

Determination of point of maximum Bending moment

$\Delta DGF \cong \Delta EFH$

$\Rightarrow \frac{DF}{DG} = \frac{EF}{EH}$

$\Rightarrow \frac{1.5 - x}{2} = \frac{x}{1}$

$\Rightarrow (1.5 - x) \times 1 = 2x$

$\Rightarrow 1.5 - x = 2x$

$\Rightarrow 1.5 = 2x + x$

$\Rightarrow 1.5 = 3x$

$\Rightarrow x = \frac{1.5}{3}$

$\Rightarrow x = 0.5$

B.M calculation

at B $= 0$

at E $= +(9 \times 1.5) - (2 \times 1.5 \times \frac{1.5}{2}) = 11.25$ KN-m

at F $= (9 \times 2) - (2 \times 2 \times \frac{2}{2}) - (5 \times 0.5) = 11.5$ KN.m

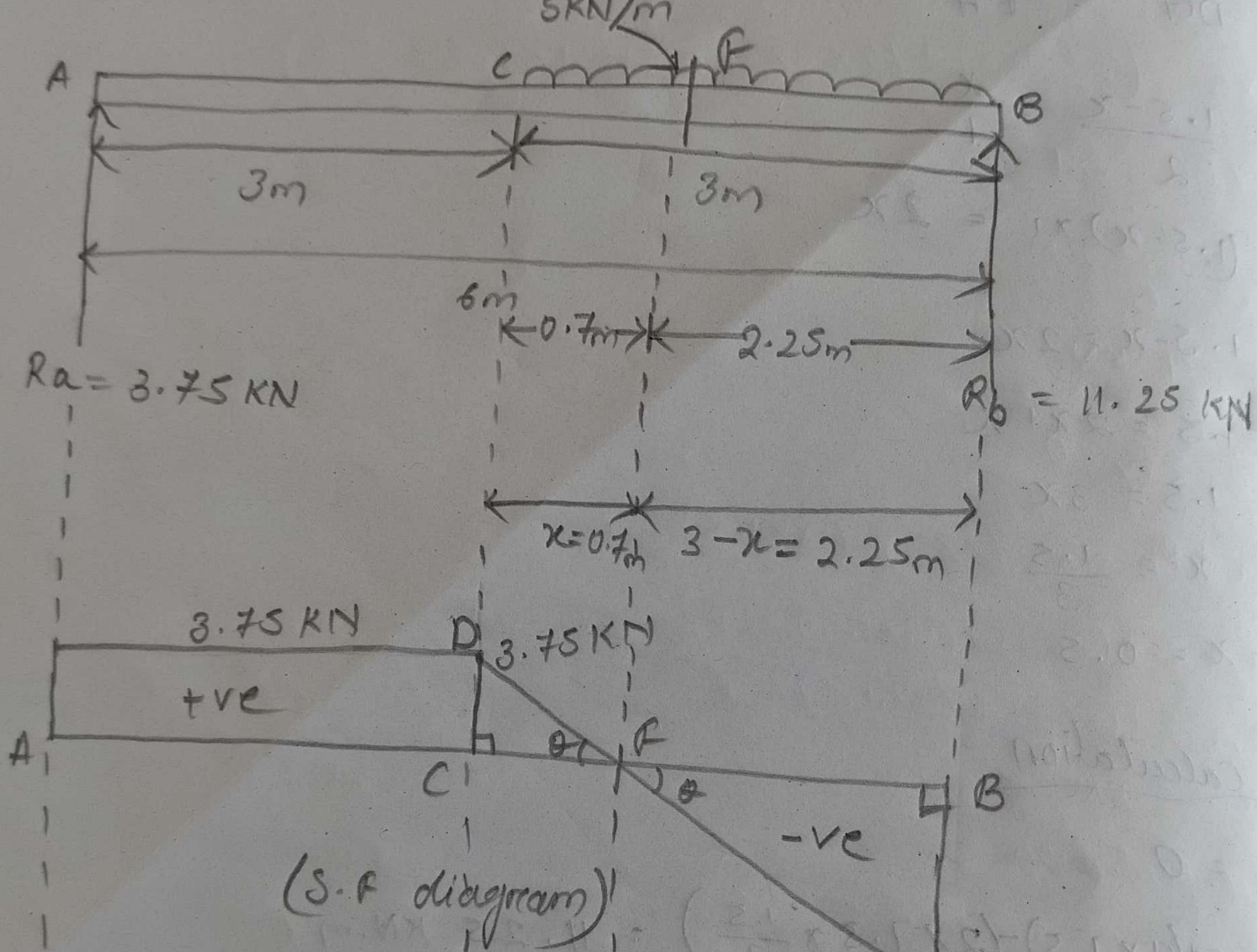
at D $= (9 \times 3) - (2 \times 3 \times \frac{3}{2}) - (5 \times 1.5) = 10.5$ KN.m

at C $= (9 \times 4.5) - (5 \times 3) - \{2 \times 3 \times (\frac{3}{2} + 1.5)\} = 7.5$ KN-m

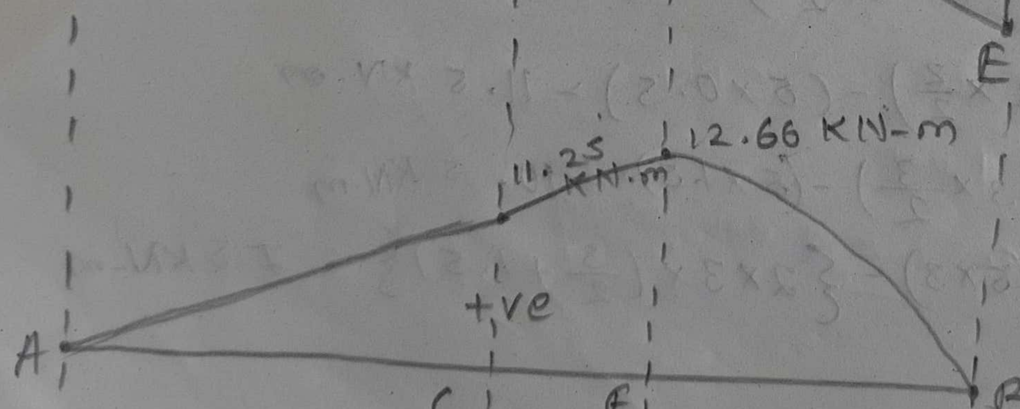
at A $= 0$

Exi:-
13.7

A simply supported beam 6 m long is carrying a uniformly distri. load of 5KN/m over a length of 3m from the right end. Draw the S.F and B.M diagram for the beam and also calculate the max B.M on the section.



(S.F diagram)



(B.M diagram)

Taking moment about 'A' point

$$\Sigma M_A = 0$$

$\Rightarrow \cancel{(R_a \times 0)}^{0} + (R_b \times 6) - (5 \times 3 \times 4.5) = 0$

$\Rightarrow R_b \times 6 = 5 \times 3 \times 4.5$

$\Rightarrow R_b \times 6 = 67.5$

$\Rightarrow R_b = \frac{67.5}{6}$

$\Rightarrow R_b = 11.25$ KN

$$\Sigma F_y = 0$$

$\Rightarrow R_a + R_b \cancel{+} - 5 \times 3 = 0$

$\Rightarrow R_a = 5 \times 3 - R_b$

$\Rightarrow R_a = 5 \times 3 - 11.25$

$\Rightarrow R_a = 3.75$ KN

S.F calculation

at B $= -11.25$ KN

up to C $= -11.25 + (5 \times 3) = 3.75$ KN

at C $= 3.75$ KN

up to A $= 3.75$ KN

at A $= 3.75$ KN

Determination of point of maximum bending moment.

$$\Delta CDF \cong \Delta FBE$$

$$\frac{CF}{CD} = \frac{BF}{BE}$$

$\Rightarrow \frac{x}{3.75} = \frac{3-x}{-11.25}$

$\Rightarrow -11.25x = (3.75)(3-x)$

$\Rightarrow -11.25x = \cancel{0}(3.75x)(11.25)$

$\Rightarrow -11.25 = 3.75x + 11.25x$

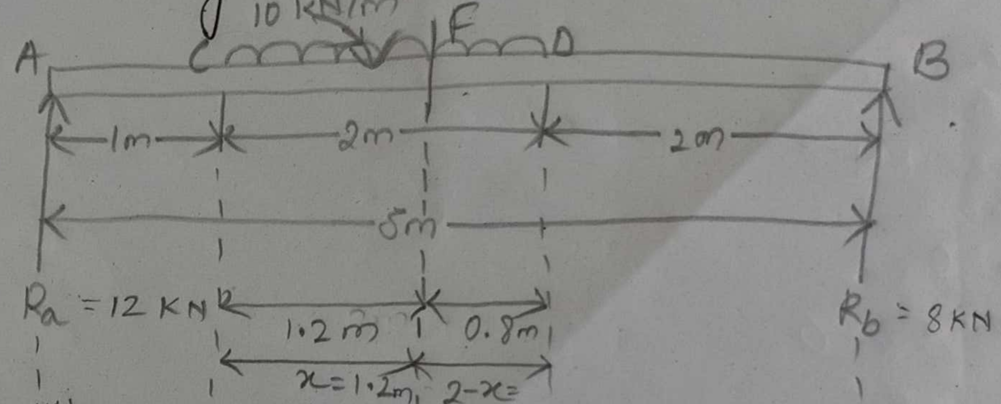
$\Rightarrow -11.25 = 15x$

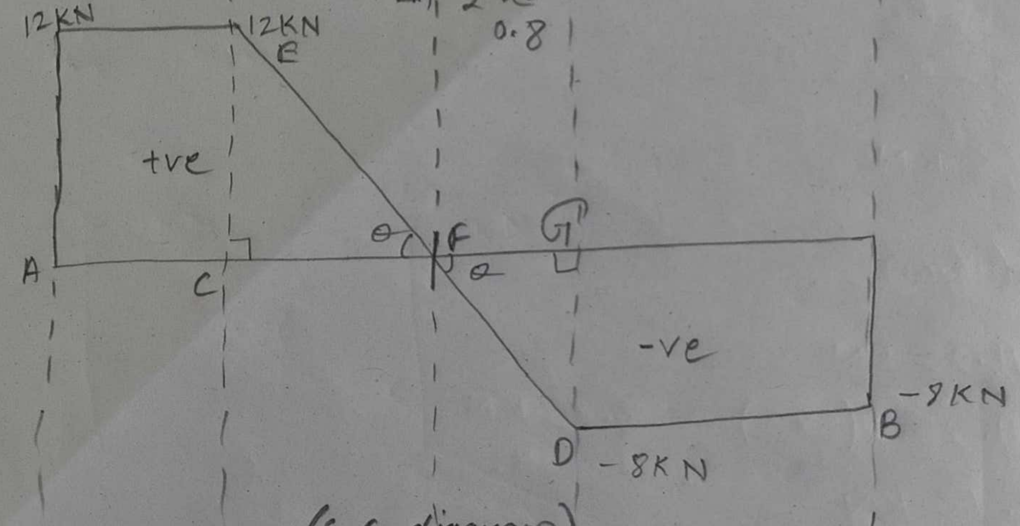
$\Rightarrow x = \frac{11.25}{15}$

$\Rightarrow x = 0.75$m

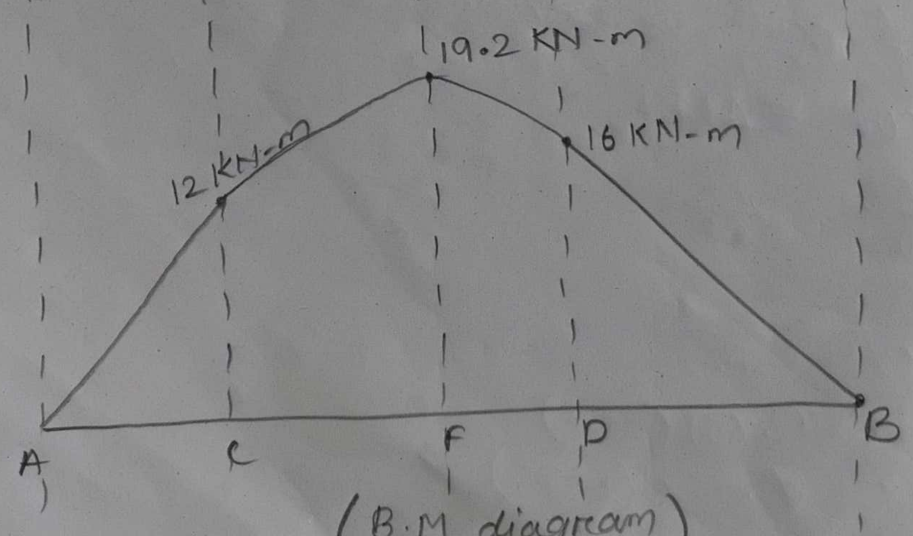
Ex:- 13.8

A simply supported beam 5m long is loaded with a uniform distributed load of 10 KN/m over a length of 2m as shown fi[...] Draw shear force and bending moment diagrams for the beam indicating the value of maximum bending moment.





(S.F diagram)



(B.M diagram)

Taking moment about 'A' point

$$\Sigma M_A = 0$$

$\Rightarrow \cancel{(R_a \times 0)}^{0} + (R_b \times 5) - (10 \times 2 \times 2) = 0$

$\Rightarrow R_b \times 5 = 10 \times 2 \times 2$

$\Rightarrow R_b \times 5 = 40$

$\Rightarrow R_b = \frac{40}{5}$

$\Rightarrow R_b = 8\ KN$

$$\Sigma F_y = 0$$

$\Rightarrow + R_a + R_b - 10 \times 2$

$\Rightarrow R_a = 10 \times 2 - R_b$

$\Rightarrow R_a = 10 \times 2 - 8$

$\Rightarrow R_a = 12\ KN$

S.F calculation

at $B = -8\ KN$

up to $D = -8\ KN$

at $D = -8\ KN$

up to $C = -8 + (10 \times 2) = 12\ KN$

at $C = 12\ KN$

up to $A = 12\ KN$

at $A = 12\ KN$

Determination of point of maximum bending moment.

$$\Delta CEF \cong \Delta FGD$$

$$\frac{CF}{CE} = \frac{GF}{GD}$$

$\Rightarrow \frac{x}{12} = \frac{2-x}{8}$

$\Rightarrow 8x = 12(2-x)$

$\Rightarrow 8x = 24 - 12x$

$\Rightarrow 8x + 12x = 24$

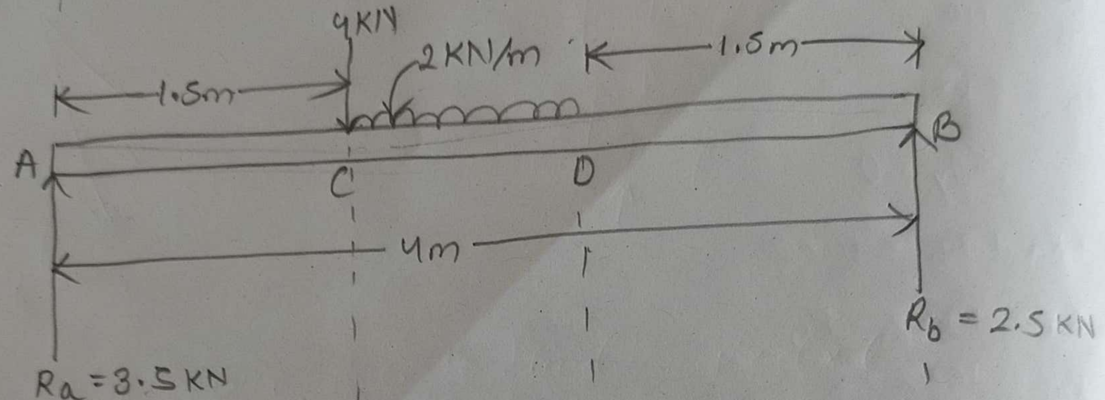
$\Rightarrow 20x = 24$

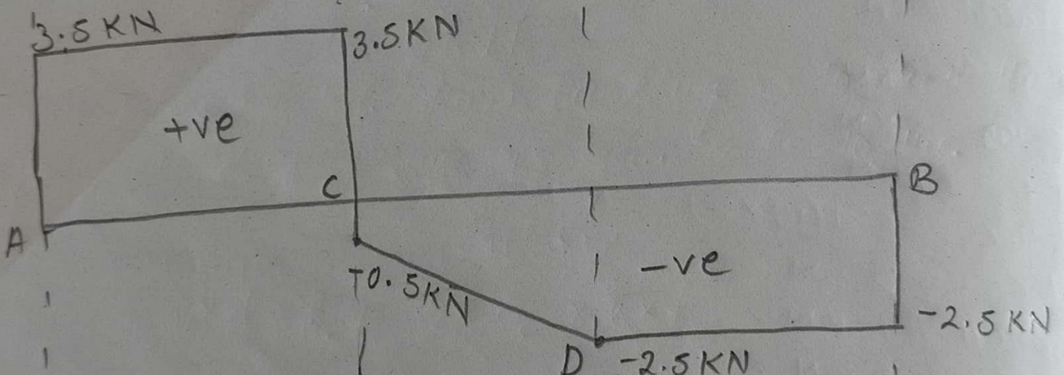
$\Rightarrow x = \frac{24}{20}$

$\Rightarrow x = 1.2\,m$

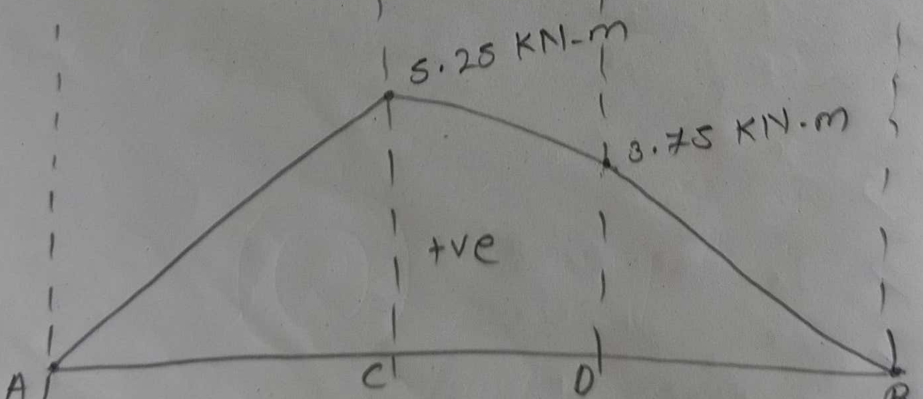
Ex:- 13.9

A simply supported beam of 4m span is carrying load as shown in Fig. Draw shear force and bending moment diagram for the beam.





(S.F diagram)



(B.M diagram)

Taking moment about 'A' point

$\Sigma M_A = 0$

$\Rightarrow \cancel{(R_a \times 0)}^{0} + (R_b \times 4) - (2 \times 1 \times 2) - (4 \times 1.5)$

$\Rightarrow R_b \times 4 = (2 \times 1 \times 2) + (4 \times 1.5)$

$\Rightarrow R_b \times 4 = 10$

$\Rightarrow R_b = \frac{10}{4}$

$\Rightarrow R_b = 2.5$ KN

$\Sigma F_y = 0$

$\Rightarrow +R_a + R_b - (2 \times 1) - 4$

$\Rightarrow R_a = (2 \times 1) + 4 - R_b$

$\Rightarrow R_a = (2 \times 1) + 4 - 2.5$

$\Rightarrow R_a = 3.5$

S.F calculation

at B = −2.5 KN

up to D = −2.5 KN

at D = −2.5 KN

up to C = −2.5 + (2×1) = −0.5 KN

at C = −0.5 + 4 = 3.5 KN

up to A = 3.5 KN

at A = 3.5 KN

The shear force change it ~~status~~ sign from −ve to +ve or vice versa at 'c' point, Hence the bending moment will be maximum at this point.

B.M calculation

at B = 0

at D = 2.5 × 1.5 = 3.75 KN-m

at C = (2.5×2.5) − (2×1×0.5) = 5.25 KN-m

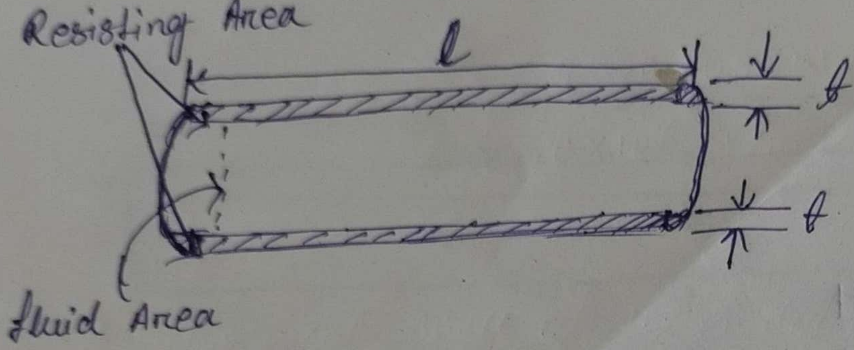
at A = 0

Point of contraflexure

It may be difind as the point where the bending moment change it's sign from −ve to +ve or vice versa.

Thin Cylindrical and Spherical Shells :-

Circumferential Stress



→ Pressure force due to fluid $= p \times d \times l$

→ Resistance force $= (\sigma \times l \times t) + (\sigma \times l \times t)$

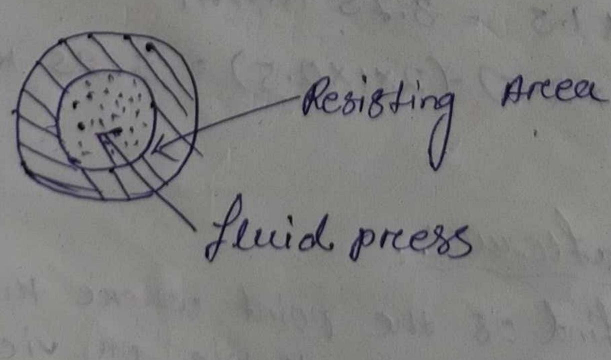
$= 2\sigma l t$

→ It will be previnted from bursting in to two through where the fluid pressure is equivalent to the resistan force.

$$p \times d \times \cancel{l} = 2 \times \sigma_c \times \cancel{l} \times t$$

$$\Rightarrow \boxed{\sigma_c = \frac{pd}{2t}}$$

Longitudinal Stress :-



→ press force due to fluid $= p \times \frac{\pi}{4} d^2$

→ The resisting force $= \sigma_l \times \pi d t$

→ It will be prevented from bursting into two cylinder where the pressure force equal to the resisting force.

$$p \times \frac{\cancel{\pi}}{4} d^{\cancel{2}} = \sigma_l \times \cancel{\pi} \cancel{d} t$$

$$\Rightarrow \sigma_l = \frac{pd}{4t}$$

Ex :- 31.1

A stream boiler of 800 mm diameter is made up of 10 mm thick plates. If the boiler is subjected to an internal pressure of 2.5 MPa, find the circumferential and longitudinal stress induced in the boiler plates.

Given :-

$d = 800 mm = 800 \times 10^{-3} m$

$t = 10 mm = 10 \times 10^{-3} m$

$p = 2.5 MPa = 2.5 \times 10^6 N/m^2$

Circumferential stress induced in the boiler plates.

we know that circumferential stress induced in the boiler plates.

$$\sigma_c = \frac{pd}{2t} = \frac{2.5 \times 10^6 \times 800 \times 10^{-3}}{2 \times 10 \times 10^{-3}}$$

$$= 100,000,000 \ N/m^2$$

$$= 100 \times 10^6 \ N/m^2$$

Longitudinal stress induced in the boiler plates :

we also know that longitudinal stress induced in the boiler plates.

$$\sigma_l = \frac{pd}{4t} = \frac{2.5\times10^6 \times 800\times10^{-3}}{4\times10\times10^{-3}}$$

$$= 50,000,000 \ N/m^2$$

$$= 50\times10^6 \ N/m^2$$

Ex :- 31.2

A cylindrical shell of 1.3 m diameter is made up of 18mm thick plates. find the circumferential and longitudinal stress in the plates, if the boiler is subjected to an internal pressure of 2.4 MPa. Take efficiency of the joints as 70

Given :-

$d = 1.3\ m$

$t = 18\ mm = 18\times10^{-3}\ m$

$p = 2.4\ MPa = 2.4\times10^6\ N/m^2$

$n = 70\% = 0.7$

circumferential stress

we know that circumferential stress,

$$\sigma_c = \frac{pd}{2tn} = \frac{2.4\times10^6 \times 1.3}{2\times18\times10^{-3}\times0.7}$$

$$= 123,809,523 \ N/m^2$$

$$= 123\times10^6 \ N/m^2$$

longitudinal stress :-

we also know that longitudinal stress,

$$\sigma_l = \frac{pd}{4t\eta} = \frac{2.4\times10^6\times1.3}{4\times18\times10^{-3}\times0.7}$$

$$= 61,904,761\ N/m^2$$

$$= 61\times10^6\ N/m^2.$$

Ex :- 31.3

A gas cylinder of internal diameter 40mm is 5mm thick. If the tensile stress in the material is not to exceed 30 MPa, find the maximum pressure which can be allowed in the cylinder.

Given :-

$d = 40mm = 40\times10^{-3}m$

$t = 5mm = 5\times10^{-3}m$

$\sigma_c = 30\ MPa = 30\times10^6\ N/m^2$

p = maximum pressure which can be allowed in the cylinder.

we know that circumferential stress (σ_c)

$$\sigma_c = \frac{pd}{2t}$$

$$\Rightarrow 30\times10^6 = \frac{p\times40\times10^{-3}}{2\times5\times10^{-3}} = 4p$$

$$\Rightarrow p = \frac{30\times10^6}{4}$$

$$= 7,500,000\ N/m^2$$

$$= 7.5\times10^6\ N/m^2$$

Note:-1.

Since the circumferential stress (σ_c) is double the longitudinal stress (σ_L), therefore in order to find the maximum pressure the given stress should be taken as circumferential stress.

Note :-2

If however, we take the given tensile stress of ~~[illegible]~~ 30×10^6 N/m^2 as the longitudial stress, then,

$$30 \times 10^6 = \frac{pd}{4t}$$

$$\Rightarrow 30 \times 10^6 = \frac{p \times 40 \times 10^{-3}}{4 \times 5 \times 10^{-3}} = 2p$$

$$\Rightarrow p = \frac{30 \times 10^6}{2}$$

$$= 15{,}000{,}000 \text{ N/m}^2$$

$$= 15 \times 10^6 \text{ N/m}^2$$

Now we shall provide a pressure of 7.5 MPa i.e (lesser of the two value) obtained by using the tensile stress as circumferential stress and longitudinal stress.

Design of Thin cylindrical shells :-

Design of thin cylindrical shell involves calculating the thickness (t) of a cylindrical shell for the given length (l), diameter (d), intensity of maximum internal pressure (p) and circumferential stress (σ_c). The required thickness of the shell is calculated from the relation.

$$t = \frac{pd}{2\sigma_c}$$

If the thickness so obtained, is not a round figure, then next higher value is provided.

Note

The thickness obtained from the longitudinal stress will be half of the thickness obtained from circumferential stress. Thus, it should not be accepted.

Ex:- 31.4

A thin cylinderical of 400mm diameter is to be designed for an internal pressure of 2.4 MPa. Find the suitable thickness of the shell, if the allowable circumferential stress is 50 MPa.

Give:-

$d = 400mm = 400 \times 10^{-3} m$

$p = 2.4 MPa = 2.4 \times 10^{6} N/m^2$

$\sigma_c = 50 MPa = 50 \times 10^{6} N/m^2$

We know that thickness of the shell.

$$t = \frac{pd}{2\sigma_c}$$

$$t = \frac{2.4 \times 10^{6} \times 400 \times 10^{-3}}{2 \times 50 \times 10^{6}}$$

$$= 9.6 \times 10^{-3} m$$

Ex:- 31.5

A cylindrical shell of 500mm diameter is required to withstand an internal pressure of 4 MPa. Find the minimum thickness of the shell, if maximum tensile strength in the plate material is 400 MPa and efficiency of the joints is 65%. Take factor of safety as 5.

Given :-

$d = 500mm = 50 \times 10^{-3} m$

$p = 4MPa = 4 \times 10^{6} N/m^2$

tensile strength $= 400 MPa = 400 \times 10^{6} N/m^2$

$\eta = 65\% = 0.65$

factor of safety = 5

We know that allowable tensile stress (i.e., circumfe stress),

$$\sigma_c = \frac{\text{Tensile Strength}}{\text{Factor of safety}}$$

$$= \frac{400 \times 10^6}{5}$$

$$= 80 \times 10^6 \ N/m^2$$

and minimum thickness of shell,

$$t = \frac{pd}{2\sigma_c \eta}$$

$$\Rightarrow t = \frac{4 \times 10^6 \times 50 \times 10^{-3}}{2 \times 80 \times 10^6 \times 0.65}$$

$$= 1.92 \times 10^{-3} m$$

Change in diameter of a thin cylindrical shell due an internal pressure.

* change in diameter,

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

* change in length,

$$\delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

where,

p = fluid pressure
d = diameter of the shell
t = Thickness of the shell
E = young' modulus
l = length of the shell
$\frac{1}{m}$ = poisson's ratio

Ex - 31.6

A cylindrical thin drum 800 mm in diameter and 4m long is made of 10mm thick plates. If the drum is subjected to an internal pressure of 2.5 MPa. determine its changes in diameter and length. Take E as 200 GPa and Poisson's ratio as 0.25.

Given

$d = 800 \text{ mm} = 800 \times 10^{-3} \text{ m}$

$l = 4 \text{ m}$

$t = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$

$p = 2.5 \text{ MPa} = 2.5 \times 10^{6} \text{ N/m}^2$

$E = 200 \text{ GPa} = 200 \times 10^{9} \text{ N/m}^2$

$\frac{1}{m} = 0.25$

change in diameter

we know change in diameter,

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{2.5 \times 10^6 \times (800 \times 10^{-3})^2}{2 \times 10 \times 10^{-3} \times 200 \times 10^9}\left(1 - \frac{0.25}{2}\right)$$

$$= 3.5 \times 10^{-4} \text{ m}$$

change in length

we also know that change in length,

$$\delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{2.5 \times 10^6 \times 800 \times 10^{-3} \times 4}{2 \times 10 \times 10^{-3} \times 200 \times 10^9}\left(\frac{1}{2} - 0.25\right)$$

$$= 5 \times 10^{-4} \text{ m}$$

Change in volume of a thin cylinder shell due to an internal pressure :-

$$\delta v = v\,(\epsilon_l + 2\epsilon_c)$$

ϵ_l = longitudinal strain $= \frac{\delta l}{l}$

ϵ_c = circumferential strain $= 2 \cdot \frac{\delta d}{d}$

$v = \frac{\pi}{4} d^2 . l$

where,

v = volume

ϵ_l = longitudinal strain

ϵ_c = circumferential strain

d = diameter

l = length.

δl = change in length

δd = change in diameter

δv = change in volume.

Ex-31.7

A cylindrical vessel 2m long and 500 mm in diameter with 10 mm thick plates is subjected to an internal pressure of 3 MPa. Calculate the change in volume of the vessel. Take E = 200 GPa and poisson's ratio = 0.3 for the vessel material.

Given

$l = 2m$

$d = 500\,mm = 500 \times 10^{-3} m$

$t = 10\,mm = 10 \times 10^{-3} m$

$p = 3\,MPa = 3 \times 10^{6}\ N/m^2$

$E = 200\,GPa = 200 \times 10^9\,N/m^2$

$\frac{1}{m} = 0.3$

we know that circumferential strain,

$$\epsilon_c = \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{3 \times 10^6 \times 500 \times 10^{-3}}{2 \times 10 \times 10^{-3} \times 200 \times 10^9}\left(1 - \frac{0.3}{2}\right)$$

$$= 3.2 \times 10^{-4}$$

longitudinal strain,

$$\epsilon_l = \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{3 \times 10^6 \times 500 \times 10^{-3}}{2 \times 10 \times 10^{-3} \times 200 \times 10^9}\left(\frac{1}{2} - 0.3\right)$$

$$= 7.5 \times 10^{-5}$$

we also know that original volume of the vessel,

$$V = \frac{\pi}{4}(d)^2 \times l$$

$$= \frac{\pi}{4}(500 \times 10^{-3})^2 \times 2$$

$$= 0.392\ m^3$$

∴ change in volume,

$$\delta V = V(\epsilon_c + 2\epsilon_l)$$

$$= 0.392\,(3.2 \times 10^{-4} + 2 \times 7.5 \times 10^{-5})\ m^3$$

$$= 1.8424 \times 10^{-4}\ m^3$$

Exercise - 31.1

1) A cylindrical shell 2m long and 1m internal diameter is made up of 20mm thick plates. Find the circumferential & longitudinal stresses in the shell material, if it is subjected to an internal pressure of 5 MPa.

Given

$l = 2m$

$d = 1m$

$t = 20mm = 20 \times 10^{-3} m$

$p = 5 MPa = 5 \times 10^{6} N/m^2$

Circumferential stress

we know that circumferential stress induced in the cylindrical shell.

$$\sigma_c = \frac{pd}{2t}$$

$$= \frac{5 \times 10^6 \times 1}{2 \times 20 \times 10^{-3}}$$

$$= 125000000 \ N/m^2$$

$$= 125 \times 10^6 \ N/m^2$$

Longitudinal stress

we also know that longitudinal stress induc in the cylindrical shell.

$$\sigma_l = \frac{pd}{4t}$$

$$= \frac{5 \times 10^6 \times 1}{4 \times 20 \times 10^{-3}}$$

$$= 62500000 \ N/m^2$$

$$= 62.5 \times 10^6 \ N/m^2$$

2) A steam boiler of 1.25 m in diameter is subjected to an internal pressure of 1.6 MPa. If the steam boiler is made up of 20 mm thick plates, calculate the circumferential and longitudinal stresses. Take efficiency of the circumferential and longitudinal joints as 75% & 60% respectively.

Given

$d = 1.25 m$

$P = 1.6 MPa = 1.6 \times 10^6 N/m^2$

$t = 20 mm = 20 \times 10^{-3} m$

$\eta_c = 75\% = 0.75$

$\eta_l = 60\% = 0.6$

circumferential stress

we know that circumferential stress

$$\sigma_c = \frac{pd}{2t\eta}$$

$$= \frac{1.6 \times 10^6 \times 1.25}{2 \times 20 \times 10^{-3} \times 0.75}$$

$$= 66,666,666.67 \; N/m^2$$

$$= 67 \times 10^6 \; N/m^2$$

longitudinal stress

we know that longitudinal stress

$$\sigma_l = \frac{pd}{4t\eta}$$

$$= \frac{1.6 \times 10^6 \times 1.25}{4 \times 20 \times 10^{-3} \times 0.6}$$

$$= 41,666,666.67 \; N/m^2$$

$$= 42 \times 10^6 \; N/m^2$$

3) A pipe of 100 mm diameter is carrying a fluid under pressure of 4 MPa. what should be the minimum thickness of the pipe. if maximum circumferential stress in the pipe material is 12.5 MPa.

Give

$d = 100mm = 100 \times 10^{-3} m$

$p = 4 MPa = 4 \times 10^6 N/m^2$

$\sigma_c = 12.5 MPa = 12.5 \times 10^6 N/m^2$

we know that thickness of the shell

$$t = \frac{pd}{2\sigma_c}$$

$$t = \frac{4 \times 10^6 \times 100 \times 10^{-3}}{2 \times 12.5 \times 10^6}$$

$$= 16 \times 10^{-3} m$$

4) A cylindrical shell 3m long has 1m internal diameter and 15mm metal thickness. calculate the circumferential and longitudinal stresses, if the shell is subjected to an internal pressure of 1.5 MPa. Also calculate the change in diamensions of the shell. Take E = 200 GPa and poisson ratio = 0.3.

Give

$l = 3m$

$d = 1m$

$t = 15mm = 15 \times 10^{-3} m$

$p = 1.5 MPa = 1.5 \times 10^6 N/m^2$

$E = 200 GPa = 200 \times 10^9 N/m^2$

$\frac{1}{m} = 0.3$

circumferential stress

we know that circumferential stress,

$$\sigma_c = \frac{pd}{2t}$$

$$= \frac{1.5 \times 10^6 \times 1}{2 \times 15 \times 10^{-3}}$$

$$= 50,000,000 \ N/m2$$

$$= 50 \times 10^6 \ N/m^2$$

longitudinal stress

we know that longitudinal stress,

$$\sigma_l = \frac{pd}{4t}$$

$$= \frac{1.5 \times 10^6 \times 1}{4 \times 15 \times 10^{-3}}$$

$$= 25,000,000 \ N/m^2$$

$$= 25 \times 10^6 \ N/m^2$$

Change in diameter

we know that change in diameter,

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{1.5 \times 10^6 \times (1)^2}{2 \times 15 \times 10^{-3} \times 200 \times 10^9}\left(1 - \frac{0.3}{2}\right)$$

$$= 2.12 \times 10^{-4} \ m$$

Change in length

we also know that change in length,

$$\delta l = \frac{pdl}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)$$

$$= \frac{1.5\times10^6\times1\times3}{2\times15\times10^{-3}\times200\times10^9}\left(\frac{1}{2}-0.3\right)$$

$$= 1.5\times10^{-4}m$$

5) A cylindrical vessel 1.8 m long 800 mm in diameter is made up of 8mm thick plates. Find the hoop and longitudinal stress in the vessel, when it contains fluid under a pressure of 2.5 MPa. Also find the changes in length, diameter and volume of the vessel. Take $E = 200$ GPa and $\frac{1}{m} = 0.3$

Given

$l = 1.8m$

$d = 800mm = 800\times10^{-3}m$

$t = 8mm = 8\times10^{-3}m$

$p = 2.5$ MPa $= 2.5\times10^6$ N/m²

$E = 200$ GPa $= 200\times10^9$ N/m²

$\frac{1}{m} = 0.3$

Circumferential stress,

we know that circumferential stress

$$\sigma_c = \frac{pd}{2t}$$

$$= \frac{2.5\times10^6\times800\times10^{-3}}{2\times8\times10^{-3}}$$

$= 125,000,000$ N/m²

$= 125\times10^6$ N/m²

Longitudinal stress,
we know longitudinal stress

$$\sigma_l = \frac{pd}{4t}$$

$$= \frac{2.5 \times 10^6 \times 800 \times 10^{-3}}{4 \times 8 \times 10^{-3}}$$

$$= 62500000 \text{ N/m}^2$$

$$= 62.5 \times 10^6 \text{ N/m}^2$$

change in diameter
we also know that change in diameter

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{2.5 \times 10^6 \times (800 \times 10^{-3})^2}{2 \times 8 \times 10^{-3} \times 200 \times 10^9}\left(1 - \frac{0.3}{2}\right)$$

$$= 4.25 \times 10^{-4} m$$

change in length
we also know that change in length

$$\delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{2.5 \times 10^6 \times 800 \times 10^{-3} \times 1.8}{2 \times 8 \times 10^{-3} \times 200 \times 10^9}\left(\frac{1}{2} - 0.3\right)$$

$$= 2.25 \times 10^{-4} m$$

Circumfarential strain,

$$\epsilon_c = \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{2.5 \times 10^6 \times 800 \times 10^{-3}}{2 \times 8 \times 10^{-3} \times 200 \times 10^9}\left(1 - \frac{0.3}{2}\right)$$

$$= 5.31 \times 10^{-4} \ N/m^2$$

$$\epsilon_l = \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{2.5 \times 10^6 \times 800 \times 10^{-3}}{2 \times 8 \times 10^{-3} \times 200 \times 10^9}\left(\frac{1}{2} - 0.3\right)$$

$$= 1.25 \times 10^{-4} \ N/m^2.$$

volume of the vessel.

$$V = \frac{\pi}{4} d^2 l$$

$$= \frac{\pi}{4}\left(800 \times 10^{-3}\right)^2 \times 1.8$$

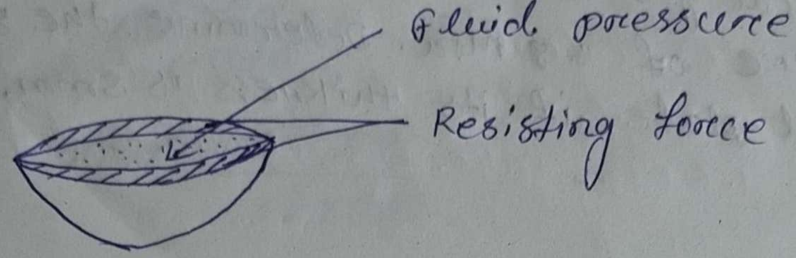
$$= 0.90 \ m^3$$

change in volume,

$$\delta V = V(\epsilon_c + 2\epsilon_l)$$

$$= 7.029 \times 10^{-4} \ m^3$$

Thin Spherical Shells



→ pressure force due to fluid $= p \times \frac{\pi}{4} d^2$

→ Resisting force $= \sigma \times \pi d t$

→ It will be prevented from bursting into two half sphere where the fluid pressure force equivalent to the resisting force.

$$p \times \frac{\cancel{\pi}}{4} d^{\cancel{2}} = \sigma \times \cancel{\pi} \cancel{d} t$$

$$\Rightarrow \boxed{\sigma = \frac{pd}{4t}}$$

$$\boxed{\sigma_l = \sigma_c = \frac{pd}{4t}}$$

Note - If η is the efficiency of the riveted joints of the spherical shell, then stress,

$$\boxed{\sigma = \frac{pd}{4t\eta}}$$

Ex - 31.8

A spherical gas vessel of 1.2 m diameter is subjected to a pressure of 1.8 MPa. Determine the stress induced in the vessel plate, if its thikness is 5mm.

Given

$d = 1.2 m$

$P = 1.8 MPa = 1.8 \times 10^6 N/m^2$

$t = 5 mm = 5 \times 10^{-3} m$

We know that stress in the vessel plates,

$$\sigma = \frac{pd}{4t}$$

$$= \frac{1.8 \times 10^6 \times 1.2}{4 \times 5 \times 10^{-3}}$$

$$= 108\,000\,000$$

$$= 108 \times 10^6 \ N/m^2$$

Ex - 31.9

A spherical vessel of 2m diameter is subjected to an internal pressure of 2MPa. Find the minimum thickness of the plates required, if the maximum stress is not to exceed 100 MPa. Take efficiency of the joint as 80%.

Given

$d = 2m$

$P = 2 MPa = 2 \times 10^6 N/m^2$

$\sigma = 100 MPa = 100 \times 10^6 N/m^2$

$\eta = 80\% = 0.8$

t = minimum thickness of the plates in m.

We know that stress in the plates (σ)

$$100\times10^6 = \frac{Pd}{4t\eta}$$

$$\Rightarrow 100\times10^6 = \frac{2\times10^6\times2}{4\times t\times0.8}$$

$$\Rightarrow 100\times10^6 = \frac{1250\times10^3}{t}$$

$$\Rightarrow t = \frac{1250\times10^3}{100\times10^6}$$

$$= 0.0125\,m$$

$$= 12.5\times10^{-3}\,m$$

Change in diameter and volume of a thin spherical shell due to an internal pressure :-

Change in diameter,

$$\delta d = \frac{Pd^2}{4tE}\left(1-\frac{1}{m}\right)$$

change in volume,

$$\delta V = \frac{\pi P d^4}{8tE}\left(1-\frac{1}{m}\right)$$

Ex - 81.10

A spherical shell of 2m diameter is made up of 10mm thick plates. Calculate the change in diameter and volume of the shell, when it is subjected to an internal pressure of 1.6 MPa. Take $E = 200$ GPa and $\frac{1}{m} = 0.3$.

Given

$d = 2m$

$t = 10mm = 10 \times 10^{-3} m$

$p = 1.6 \text{ MPa} = 1.6 \times 10^{6} \text{ N/m}^2$

$E = 200 \text{ GPa} = 200 \times 10^{9} \text{ N/m}^2$

$\frac{1}{m} = 0.3$

Change in diameter

we know that change in diameter,

$$\delta d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)$$

$$= \frac{1.6 \times 10^6 \times (2)^2}{4 \times 10 \times 10^{-3} \times 200 \times 10^9}(1 - 0.3)$$

$$= 5.6 \times 10^{-4} m$$

Change in volume

we know that change in volume,

$$\delta v = \frac{\pi p d^4}{8tE}\left(1 - \frac{1}{m}\right)$$

$$= \frac{\pi \times 1.6 \times 10^6 \times (2)^4}{8 \times 10 \times 10^{-3} \times 200 \times 10^9}(1 - 0.3)$$

$$= 3.518 \times 10^{-3} m^3$$

Bending Stresses in Simple Beams :-

Bending Stress :-

Consider a small length dx of a beam subjected to a bending moment as shown in fig.(a). As a result of this moment, let this small length of beam bend into an arc of a circle with O as centre as shown in fig. (b)

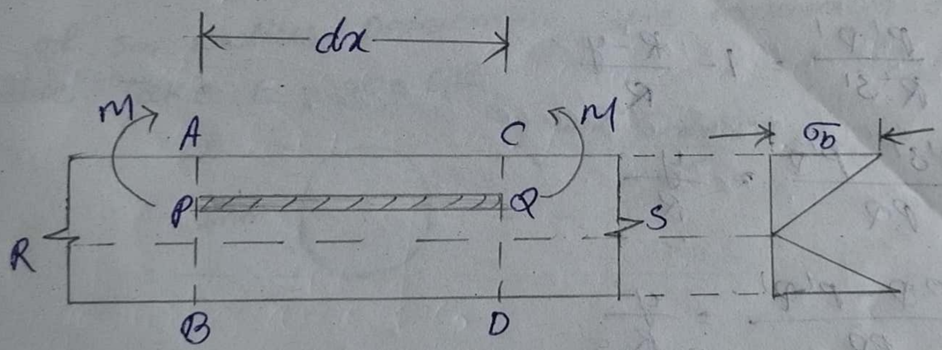
Let

M = moment acting at the beam

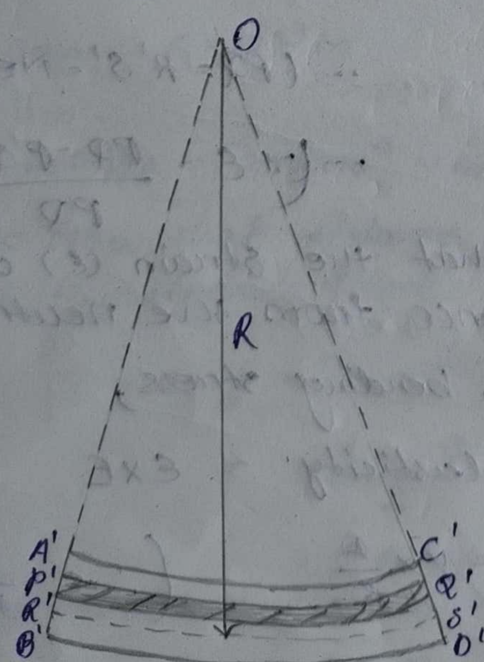
θ = Angle subtended at the centre by the arc and

R = Radius of curvature of the beam.

(a)



(b)



Bending Stress

Now consider a layer PQ at a distance y from RS the neutral axis of the beam. Let this layer be compressed to P'Q' after bending as shown in fig.(b)

We know that decrease in length of this layer,

$$\delta l = PQ - P'Q'$$

$$\therefore \text{strain } \varepsilon = \frac{\delta l}{\text{Original length}} = \frac{PQ - P'Q'}{PQ}$$

Now from the geometry of the curved beam, we find that the two sections OP'Q' and OR'S' are similar.

$$\therefore \frac{P'Q'}{R'S'} = \frac{R-y}{R}$$

$$\text{or } 1 - \frac{P'Q'}{R'S'} = 1 - \frac{R-y}{R}$$

$$\text{or } \frac{R'S' - P'Q'}{PQ} = \frac{y}{R}$$

$$\frac{PQ - P'Q'}{PQ} = \frac{y}{R}$$

$$\varepsilon = \frac{y}{R}$$

... (PQ = R'S' = Neutral axis)

$$\ldots \left(\because \varepsilon = \frac{PQ - P'Q'}{PQ}\right)$$

It is thus obvious, that the strain (ε) of a layer is proportinal to its distance from the neutral axis.

We also know that the bending stress,

$$\sigma_b = \text{strain} \times \text{Elasticity} = \varepsilon \times E$$

$$= \frac{y}{R} \times E = y \times \frac{E}{R} \qquad \left(\because \varepsilon = \frac{y}{R}\right)$$

Since E and R are constants in this expression, therefore the stress at any point is directly proportional to y, i.e., the distance of the point from the neutral axis. The above expression may also be written as,

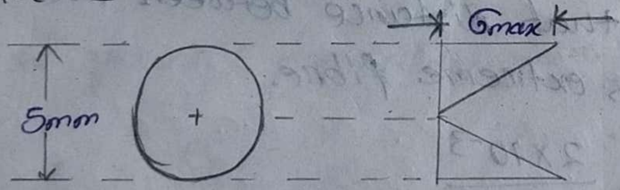
$$\frac{\sigma_b}{y} = \frac{E}{R} \text{ or } \sigma_b = \frac{E}{R} \times y$$

<u>Note</u>

Since the bending stress in inversely proportional to the radius (R), therefore for maximum stress the radius should be minimum and vice versa.

<u>Ex- 14.1</u>

A steel wire of 5mm diameter is bent into a circular shape of 5m radius. Determine the maximum stress induced in the wire. Take E = 200 GPa.



Given

$d = 5mm = 5 \times 10^{-3} m$

$R = 5m$

$E = 200 GPa = 200 \times 10^{9} N/m^2$

We Now that distance between the neutral axis of the wire and its extreme fibre.

$$y = \frac{d}{2}$$

$$= \frac{5 \times 10^{-3}}{2}$$

$$= 2.5 \times 10^{-3}$$

and maximum bending stress induced in the wire.

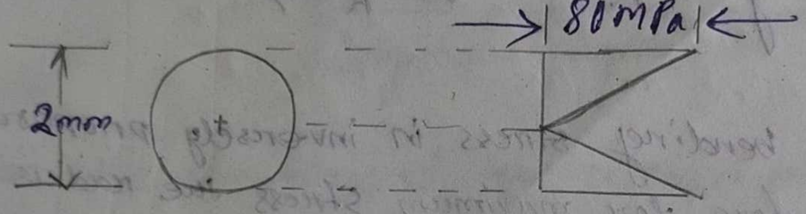
$$\sigma_{b\,max} = \frac{E}{R} \times y$$

$$= \frac{200 \times 10^{9}}{5} \times 2.5 \times 10^{-3} = 100000000$$

$$= 100 \times 10^{6} N/m^2$$

Ex - 14.2

A copper wire of 2 mm diameter is required to be wound around a drum. Find the minimum radius of the drum, if the stress in the wire is not to exceed 80 MPa. Take modulus of elasticity for the copper as 100 GPa.



Given

$d = 2mm = 2 \times 10^{-3} m$

$\sigma_{b(max)} = 80 \text{ MPa} = 80 \times 10^{6} \text{ N/m}^2$

$E = 100 \text{ GPa} = 100 \times 10^{9} \text{ N/m}^2$

We know that distance between the neutral axis of the wire and its extreme fibre.

$$y = \frac{2 \times 10^{-3}}{2}$$

$$= 1 \times 10^{-3}$$

∴ minimum radius of the drum

$$R = \frac{y}{\sigma_{b max}} \times E$$

$$= \frac{1 \times 10^{-3}}{80 \times 10^{6}} \times 100 \times 10^{9} \qquad \left(\because \frac{\sigma_b}{y} = \frac{E}{R} \right)$$

$$= 1.25 \text{ m}$$

Ex - 14.3

A metallic rod of 10mm diameter is bent into a circular form of radius 6 m. If the maximum bending stress developed in the rod is 125 MPa, find the value of young's modulus for the rod material.

Given

$d = 10mm = 10\times10^{-3}m$

$R = 6m$

$\sigma_{b(max)} = 125 MPa = 125\times10^{6} N/m^2$

We know that distance between the neutral axis of the rod and its extreme fibre,

$$y = \frac{10\times10^{-3}}{2}$$

$$= 5\times10^{-3}$$

$\therefore$ value of young's modulus for the rod material,

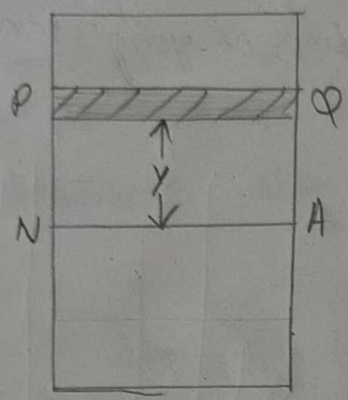
$$E = \frac{\sigma_b}{y}\times R \qquad \left(\because \frac{\sigma_b}{y} = \frac{E}{R}\right)$$

$$= \frac{125\times10^6}{5\times10^{-3}}\times6$$

$$= 1.5\times10^{11}$$

$$= 150\times10^{9} N/m^2$$

Moment of Resistance



We have already see in Art 14.2 that on one side of the neutral axis there are compressive stresses and on the other there are tensile stresses. These stresses form a couple, whose moment must be equal to the external moment (M). The moment of this couple, which resists the external bending moment, is known as moment of resistance.

Consider a section of the beam as shown in fig. Let NA be the neutral axis of the section. Now consider a small layer PQ of the beam section at a distance y from the neutral axis as shown in fig.

Let

δa = Area of the layer PQ

we have seen in Art. that the intensity of stress in the layer PQ.

$$\sigma = y \times \frac{E}{R}$$

$\therefore$ Total stress in the layer PQ

$$= y \times \frac{E}{R} \times \delta a$$

and moment of this total stress about the neutral axis

$$= y \times \frac{E}{R} \times \delta a \times y$$

$$= \boxed{\frac{E}{R} y^2 \delta a}$$

The algebraic sum of all such moment about the neutral axis must be equal to M. Therefore.

$$M = \Sigma \frac{E}{R} y^2 \cdot \delta a$$

$$= \boxed{\frac{E}{R} \Sigma y^2 \cdot \delta a}$$

The expression $\Sigma y^2 \cdot \delta a$ represents the moment of inertia of the area of the whole section about the neutral axis. Therefore

$$M = \frac{E}{R} \times I \quad \text{(where } I = \text{moment of inertia)}$$

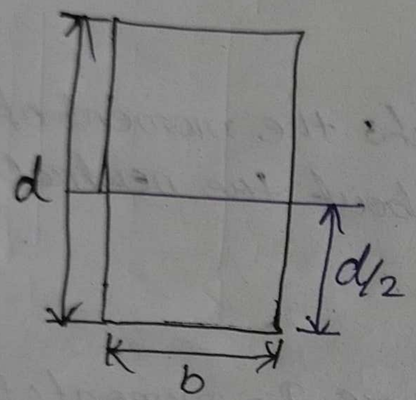
or $\frac{M}{I} = \frac{E}{R}$

we have already see in Art that,

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\therefore \boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

It is the most important equation in the theory of simple bending, which gives us relation between various characteristics of a beam.



$Z = I/y$

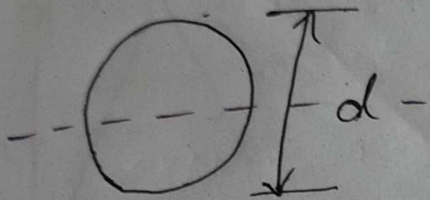
$I_x = \frac{bd^3}{12}$

$y_{max} = \frac{d}{2}$

$Z = \frac{\left(\frac{bd^3}{12}\right)}{(d/2)}$

$= \left(\frac{bd^{\cancel{3}\,2}}{\cancel{12}\,6}\right) \times \left(\frac{\cancel{2}}{\cancel{d}}\right)$

$= \frac{bd^2}{6}$



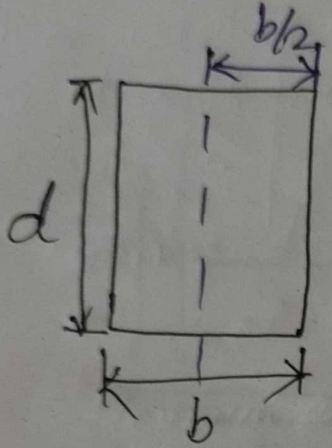
$$I_x = \frac{\pi d^4}{64}$$

$$Y_{max} = \frac{d}{2}$$

$$Z = \frac{\left(\frac{\pi d^4}{64}\right)}{\left(\frac{d}{2}\right)}$$

$$= \left(\frac{\pi d^{\cancel{4}\,3}}{\cancel{64}_{32}}\right) \times \left(\frac{\cancel{2}}{\cancel{d}}\right)$$

$$= \frac{\pi d^3}{32}$$



$$I_y = \frac{db^3}{12}$$

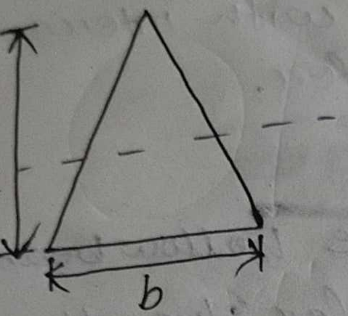
$$x_{max} = b/2$$

$$Z = \frac{I}{y}$$

$$= \frac{\left(\frac{db^3}{12}\right)}{(b/2)}$$

$$= \left(\frac{db^{\cancel{3}\,2}}{\cancel{12}_{\,6}}\right) \times \left(\frac{\cancel{2}}{\cancel{b}}\right)$$

$$= \frac{db^2}{6}$$



$$I_x = \frac{bh^3}{12}$$

$$Y_{max} = \frac{2h}{3}$$

$$Z = \frac{I}{Y}$$

$$= \frac{\left(\frac{bh^3}{12}\right)}{\left(2h/3\right)}$$

$$= \left(\frac{bh^{\cancel{3}}}{\cancel{12}_{\,4}}\right) \times \left(\frac{\cancel{3}}{2\cancel{h}}\right)$$

$$= \frac{bh}{4}$$

Ex - 14.6

Two beams are simply supported over the same span and have the same flexural strength. Compare the weights of these two beams, if one of them is solid and the other is hollow circular with internal diameter half of the external diameter.

Given

span of the solid beam = span of the hollow beam

flexural strength of solid beam = flexular strength of the hollow section.

where,

D_s = Diameter of the solid beam

D_H = Diameter of the hollow beam.

Equal span $L_s = L_H = L$

Equal flexural rigidity

$$\frac{I_s}{Y_s} = \frac{I_H}{Y_H}$$

$$\Rightarrow \frac{\left(\frac{\pi D_s^4}{64}\right)}{\left(\frac{D_s}{2}\right)} = \frac{\frac{\pi}{64}\left(D_H^4 - d_H^4\right)}{\left(\frac{D_H}{2}\right)}$$

$$\Rightarrow \frac{\cancel{\pi} D_s^{\cancel{4}}}{\cancel{64}} \times \frac{\cancel{2}}{\cancel{D_s}} = \frac{\cancel{\pi}}{\cancel{64}} (D_H^4 - d_H^4) \times \frac{\cancel{2}}{D_H}$$

$$\Rightarrow D_s^3 = \frac{D_H^4 - \left(\frac{D_H}{2}\right)^4}{D_H}$$

$$\Rightarrow D_s^3 = \frac{\frac{7}{8} D_H^{\cancel{4}}}{\cancel{D_H}}$$

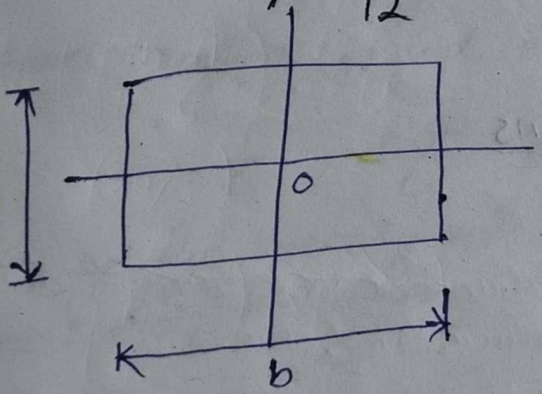
$$\Rightarrow D_s^3 = \frac{7}{8} D_H^3$$

$$\Rightarrow D_s = \sqrt[3]{\frac{7}{8} D_H}$$

$$\Rightarrow D_s = 0.956\, D_H$$

Polar moment of Inertia :- polar moment of Inertia in rectangular section.

$$I_y = \frac{hb^3}{12}$$



$$I_x = \frac{bh^3}{12}$$

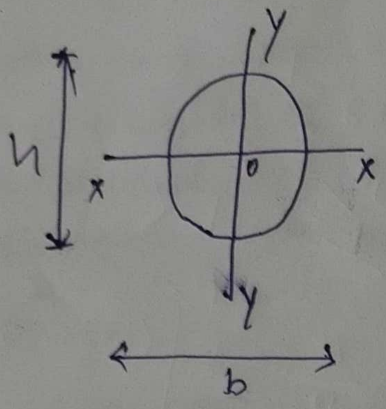
$$J = I_x + I_y$$

$$= \frac{bh^3}{12} + \frac{hb^3}{12}$$

$$= \frac{1}{12} bh(h^2 + b^2)$$

$$= \frac{bh}{12}(h^2 + b^2)$$

★ Polar moment of Inertia in circular section



$$J = I_x + I_y$$

$$= \frac{\pi d^4}{64} + \frac{\pi d^4}{64}$$

$$= 2 \cdot \frac{\pi d^4}{64}$$

$$= \frac{\pi d^4}{32}$$

Torsional stresses and strains

consider a circular shaft fixed at one end and subjected to a torque at the other end as shown in fig.

T = Torque in N-mm.

l = Length of the shaft in mm and

R = Radius of the circular shaft in mm.

As a result of this torque, every cross-section of the shaft will be subjected to shear stresses. Let the line CA on the surface of the shaft be deformed to CA' and OA to

OA' as shown in fig.

Let $\angle ACA' = \phi$ in degrees

$\angle AOA' = \theta$ in radians

τ = shear stress induced at the surface and

C = Modulus of rigidity. also known as torsional rigidity of the shaft material

we know that shear strain = Deformation per unit length

$$= \frac{AA'}{l} = \tan\theta$$

$$= \phi$$

we also know that the arc $AA' = R.\theta$

$$\therefore \phi = \frac{AA'}{l} = \frac{R.\theta}{l}$$

If τ is the intensity of shear stress on the outermost layer and C the modulus of rigidity of the shaft, then

$$\phi = \frac{\tau}{C}$$

from equations (i) and (ii), we find that

$$\frac{\tau}{C} = \frac{R.\theta}{l} \quad \text{or} \quad \frac{\tau}{R} = \frac{C\theta}{l}$$

If τ_x be the intensity of shear stress, on any layer at a distance x from the centre of the shaft, then

$$\frac{\tau_x}{x} = \frac{\tau}{R} = \frac{C.\theta}{l} \quad \text{or} \quad \frac{T}{J} = \frac{C.\theta}{l} = \frac{\tau}{R}$$

Strength of a Solid shaft

$$\boxed{T = \frac{\pi}{16}\cdot\tau\cdot D^3}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\Rightarrow \frac{T}{\left(\frac{\pi d^4}{32}\right)} = \frac{\tau}{\left(\frac{d}{2}\right)}$$

$$\Rightarrow \frac{32T}{\pi d^4} = \frac{2.\tau}{d}$$

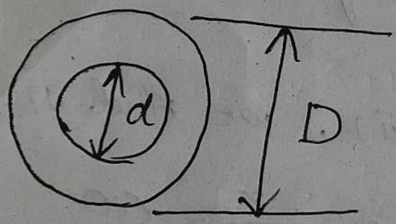
$$\Rightarrow 32T = \frac{2.\tau}{d} \times \pi d^4$$

$$\Rightarrow T = \frac{\cancel{2}.\tau}{\cancel{d}} \times \frac{\pi d^{\cancel{4}\,3}}{\cancel{32}\,16}$$

$$\Rightarrow \boxed{T = \frac{\pi}{16} . \tau . d^3}$$

Strength of a hollow circular shaft :-

$$T = \frac{\pi}{16} . \tau \left(\frac{D^4 - d^4}{D} \right)$$



Ex-27.1

A circular shaft of 50mm diameter is required to transmit torque from one shaft to another. Find the safe torque, which the shaft can transmit, if the shear stress is not to exceed 40MPa

Given

$D = 50mm = 50 \times 10^{-3} m$

$\tau = 40 \text{ MPa} = 40 \times 10^6 N/m^2$

we know that the safe torque, which the shaft can transmit

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 40 \times 10^6 \times (50 \times 10^{-3})^3$$

$$= 981.74 \text{ N-m}$$

Exi-27.2

A solid steel shaft is to transmit a torque of 10 KN-m. If the shearing stress is not to exceed 45 MPa, find the minimum diameter of the shaft.

Given

$T = 10\ KN\text{-}m = 10 \times 10^3\ N\text{-}m$

$\tau = 45\ MPa = 45 \times 10^6\ N/m^2$

Let

D = minimum diameter of the shaft in mm.

we know that torque transmitted by the shaft (T),

$$10 \times 10^3 = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 45 \times 10^6 \times D^3$$

$$= 8.835 \times 10^6 D^3$$

$$D^3 = \frac{10 \times 10^3}{8.835 \times 10^6}$$

$$= 1.132 \times 10^{-3}$$

$$D = 1.043 \times 10^{-3}$$

Ex-27.3

A hollow shaft of external and internal diameter of 80mm and 50mm is required to transmit torque from one end to the other. what is the safe torque it can transmit, if the allowable shear stress is 45 MPa?

Given

$D = 80mm = 80 \times 10^{-3} m$

$d = 50mm = 50 \times 10^{-3} m$

$\tau = 45\ MPa = 45 \times 10^6\ N/m^2$

we know that torque transmitted by the shaft.

$$T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D} \right)$$

$$\cancel{= \frac{\pi}{16} \times 45 \times 10^6 \times (80 \times 10^{-3})^4}$$

$$= \frac{\pi}{16} \times 45 \times 10^6 \times \left\{ \frac{(80 \times 10^{-3})^4 - (50 \times 10^{-3})^4}{80 \times 10^{-3}} \right\}$$

$= 3,833.60$ N-m

Ex:- 27.4

A circular shaft of 60 mm diameter is running at 150 r.p.m If the shear stress is not to exceed 50 MPa find the power which can be transmitted by the shaft.

Given

$D = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$

$N = 150$ r.p.m

$\tau = 50 \text{ MPa} = 50 \times 10^6 \text{ N/m}^2$

we know that torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 50 \times 10^6 \times (60 \times 10^{-3})^3$$

$= 2,120$ N-m

and power which can be transmitted by the shaft,

$$P = \frac{2\pi N T}{60}$$

$$= \frac{2\pi \times 150 \times 2,120}{60}$$

$= 33,300$ W

Ex:-27.5

A hollow shaft of external & internal diameters as 100 mm & 40 mm is transmitting power at 120 r.p.m find the power the shaft can transmit, if the shearing stress is not to exceed 50 MPa.

Given

$D = 100 mm = 100 \times 10^{-3} m$

$d = 40 mm = 40 \times 10^{-3} m$

$N = 120$ r.p.m

$\tau = 50 MPa = 50 \times 10^{6} N/m^2$

we know that torque the shaft can transmit.

$$T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$$

$$= \frac{\pi}{16} \times 50 \times 10^6 \times \left\{\frac{(100 \times 10^{-3})^4 - (40 \times 10^{-3})^4}{100 \times 10^{-3}}\right\}$$

$$= 9566 \text{ N-m}$$

and power the shaft can transmit,

$$P = \frac{2\pi N T}{60}$$

$$= \frac{2\pi \times 120 \times 9566}{60} = 120,209 \text{ w}$$

Ex-27.6

A solid circular shaft of 100 mm diameter is transmitting 120 KW at 150 r.p.m find the intensity of shear stress in the shaft.

Given

$D = 100 mm = 100 \times 10^{-3} m$

$P = 120$ KW

$N = 150$ r.p.m

T = Torque transmitted by the shaft
τ = Intensity of shear stress in the shaft.

we know that power transmitted by the shaft (P).

$$120 = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 150 \times T}{60}$$

$$= 15.70\,T$$

$$\Rightarrow T = \frac{120}{15.70} = 7.643$$

we also know that torque transmitted by the shaft

$$7.643 = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times \tau \times (100 \times 10^{-3})^3$$

$$= 1.96 \times 10^{-4}\,\tau$$

$$\tau = \frac{7.643}{1.96 \times 10^{-4}}$$

$$= 38,994.89$$

Ex-27.7

A hollow shaft is to transmit 200 KW at 80 r.p.m If shear stress is not to exceed 60 MPa and internal is 0. the external diameter, find the diameter of the shaft

Given

$P = 200$ KW
$N = 80$ r.p.m
$\tau = 60$ MPa $= 60 \times 10^6$ N/m^2
$d = 0.6$

we know that torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$$

$$= \frac{\pi}{16} \times 60 \times 10^6 \times \left(\frac{\cancel{D^4}^{3} - (0.6)^4}{\cancel{D}}\right)$$

$$= 1.52 \times 10^6\, D^3$$

we also know that power transmitted by the shaft (P),

$$200 = \frac{2\pi N T}{60}$$

$$= \frac{2\pi \times 80 \times 1.52 \times 10^6 D^3}{60}$$

$$= 12733\,922$$

$$= 1.273 \times 10^{13} D^3$$

$$D^3 = \frac{200}{1.273 \times 10^{13}}$$

$$= 1.572 \times 10^{-11}$$

$$D = 1.162 \times 10^{-11}$$

$$d = 0.60$$

$$= 0.6 \times 1.162 \times 10^{-11}$$

$$= 6.972 \times 10^{-12}$$

Ex - 27.8

A solid steel shaft has to transmit 100 kW at 160 r.p.m Taking allowable shear stress as 70 MPa, find the suitable diameter of the shaft. The maximum torque transmitted in each revolution exceed the mean by 20%..

Given $P = 100$ kW

$N = 160$ r.p.m

$\tau = 70$ MPa $= 70 \times 10^6$ N/m²

$T_{max} = 1.2T$

we know that power transmitted by shaft (P),

$$100 = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 160 \times T}{60} = 16.75\,T$$

$$\Rightarrow T = \frac{100}{16.75} = 5.970 \text{ N-m}$$

maximum torque

$$T_{max} = 1.2T$$

$$= 1.2 \times 5.970$$

$$= 7.164 \text{ N-m}$$

we also know that maximum torque.

$$7.164 = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 70 \times 10^6 \times D^3 = 13.74 \times 10^6 D^3$$

$$\Rightarrow D^3 = \frac{7.164}{13.74 \times 10^6} = 5.214 \times 10^{-7}$$

$$D = 1.73 \times 10^{-7}$$

Ex - 27.9

Calculate the maximum torque that a shaft of 125mm diameter can transmit, if the maximum angle of twist is 1 in a length of 1.5m. Take $C = 70$ GPa.

Given

$D = 125\text{ mm} = 125 \times 10^{-3}\text{ m}$

$\theta = 1^\circ = \frac{\pi}{180}$

$l = 1.5\text{ m}$

$C = 70\text{ GPa} = 70 \times 10^9 \text{ N/m}^2$

We know that polar moment of inertia of a solid circular shaft,

$$J = \frac{\pi}{32} \times (D)^4$$

$$= \frac{\pi}{32} \times (125 \times 10^{-3})^4$$

$$= 2.39 \times 10^{-5}$$

and relation for torque transmitted by the shaft.

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{T}{2.39 \times 10^{-5}} = \frac{70 \times 10^9 \times (\pi/180)}{1.5} = 814 \times 10^6$$

$$T = 814 \times 10^6 \times 2.39 \times 10^{-5}$$

$$= 19,454 \text{ N-m}$$

Ex:- 27.10

Find the angle of twist per metre length of a hollow shaft of 100 mm external and 60 mm internal diameter, if the shear stress is not to exceed 35 MPa. Take $C = 85$ GPa.

Given, $l = 1m$.

$D = 100mm = 100 \times 10^{-3} m$

$d = 60mm = 60 \times 10^{-3} m$

$\tau = 35 \text{ MPa} = 35 \times 10^6 \text{ N/m}^2$

$C = 85 \text{ GPa} = 85 \times 10^9 \text{ N/m}^2$

$\theta = ?$

We know that torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$$

$$= \frac{\pi}{16} \times 35 \times 10^6 \times \left\{\frac{(100 \times 10^{-3})^4 - (60 \times 10^{-3})^4}{100 \times 10^{-3}}\right\}$$

$$= 5.98 \times 10^{11} \text{ N-m}$$

we also know that polar moment of inertia of a hollow circular shaft,

$$J = \frac{\pi}{32}(D^4 - d^4)$$

$$= \frac{\pi}{32}\left\{(100 \times 10^{-3})^4 - (60 \times 10^{-3})^4\right\}$$

$$= 8.55 \times 10^{-6}$$

and relation for the angle of twist,

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{5.98 \times 10^4}{8.55 \times 10^{-6}} = \frac{85 \times 10^9 \times \theta}{1} = 8.5 \times 10^{10}\theta$$

$$\theta = \frac{5.98 \times 10^4}{8.55 \times 10^{-6} \times 8.5 \times 10^{10}} = 0.082 = 4.7^\circ$$

Ex - 27.11

A solid shaft of 120 mm diameter is required to transmit 200 KW at 100 r.p.m. If the angle of twist not to exceed 2°, find the length of the shaft. Take modulus of rigidity for the shaft material as 90GPa.

Given

$D = 120mm = 120 \times 10^{-3} m$

$P = 200$ KW

$N = 100$ R.P.m

$\theta = 2^\circ = \frac{2\pi}{180}$

$C = 90$ GPa $= 90 \times 10^9$ N/m²

$T = ?$

$l = ?$

we know that power transmitted by the shaft (P.)

$$200 = \frac{2\pi NT}{60} = \frac{2\pi \times 100 \times T}{60} = 10.47T$$

$$T = \frac{200}{10.47} = 19.10 \text{ N-m}$$

we also know that polar moment of inertia of a solid shaft,

$$J = \frac{\pi}{32} \times (D)^4$$

$$= \frac{\pi}{32} \times (120 \times 10^{-3})^4 = 2.04 \times 10^{-5}$$

relation for the length of the shaft.

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{19.10}{2.04 \times 10^{-5}} = \frac{90 \times 10^9 \times 2\pi/180}{l}$$

$$0.936 \times 10^6 = \frac{3.14 \times 10^9}{l}$$

$$l = \frac{3.14 \times 10^9}{0.936 \times 10^6} = 3354.70$$

Ex-27.12

Find the maximum torque, that can be safely applied to a shaft of 80 mm diameter. The permissible angle of twist is 1.5 degree in a length of 5m and shear stress not to exceed 42 MPa - Take C = 84 GPa.

Given

$D = 80\text{mm} = 80 \times 10^{-3} \text{m}$

$\theta = 1.5^\circ = \frac{1.5\pi}{180}$ rad.

$l = 5\text{m}$

$\tau = 42 \text{ MPa} = 42 \times 10^6 \text{ N/m}^2$

$C = 84 \text{ GPa} = 84 \times 10^9 \text{ N/m}^2$

1) Torque based on shear stress

we know that the torque which can be applied to the shaft.

$$T_1 = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 42 \times 10^6 \times (80 \times 10^{-3})^3$$

$$= 4.22 \times 10^3 \text{ N-m}$$

2) Torque based on angle of twist

we also know that polar moment of inertia of a solid circu shaft.

$$J = \frac{\pi}{32} (D)^4$$

$$= \frac{\pi}{32} \times (80 \times 10^{-3})^4$$

$$= 4.02 \times 10^{-6}$$

relation for the torque that can be applied.

$$\frac{T_2}{J} = \frac{C.\theta}{l}$$

$$\Rightarrow \frac{T_2}{4.02 \times 10^{-6}} = \frac{84 \times 10^9 . 1.5\pi/180}{5} = 0.43 \times 10^9$$

$$\Rightarrow T_2 = 0.43 \times 10^9 \times 4.02 \times 10^{-6} = 1728.6 \text{ N-m}$$

Ex - 27.13

A solid shaft is subjected to a torque of 1.6 KN-m. Find the necessary diameter of the shaft, if the allowable shear stress is 60 MPa. The allowable twist is 1° for every 20 diameters length of the shaft. Take C = 80 GPa.

Given

$T = 1.6$ KN-m $= 1.6 \times 10^3$ N-m

$\tau = 60$ MPa $= 60 \times 10^6$ N/m²

$\theta = 1° = \frac{\pi}{180}$ rad

$l = 20D$

$C = 80$ GPa $= 80 \times 10^9$ N/m²

1) Diameter for strength

we know that torque transmitted by the shaft (T),

$$1.6 \times 10^3 = \frac{\pi}{16} \times \tau \times D_1^3$$

$$= \frac{\pi}{16} \times 60 \times 10^6 \times D_1^3 = 11.78 \times 10^6 D_1^3$$

$$D_1^3 = \frac{1.6 \times 10^3}{11.78 \times 10^6} = 1.36 \times 10^{-4}$$

$$D_1 = 1.10 \times 10^{-4} \text{ m}$$

2) Diameter for stiffness

we know that polar moment of inertia of a solid circular shaft,

$$J = \frac{\pi}{32} \times (D_2)^4 = 0.098 D_2^4$$

relation for the diameter,

$$\frac{T}{J} = \frac{C \cdot \theta}{l}$$

$$\frac{1.6 \times 10^3}{0.098 D_2^4} = \frac{80 \times 10^9 \times \pi/180}{20 D_2}$$

$$D_2^3 = \frac{1.6 \times 10^3 \times 20}{0.098 \times 80 \times 10^9 \times \pi/180} = 2.34 \times 10^{-4}$$

$$D_2 = 1.33 \times 10^{-4}$$

Ex-27.14

A solid shaft of 200mm diameter has the same cross-sectional area as a hollow shaft of the same material with inside diam of 150mm. Find the ratio of

a) powers transmitted by both the shafts at the same angular velocity.

b) angles of twist in equal lengths of these shafts, when stressed to the same intensity.

Given

$D_1 = 200mm = 200 \times 10^{-3} m$

$d = 150mm = 150 \times 10^{-3} m$

a) Ratio of powers transmitted by both the shaft,

$$A_1 = \frac{\pi}{4} \times D_1^2$$

$$= \frac{\pi}{4} \times (200 \times 10^{-3})^2 = 0.01\pi \; m^2$$

cross-sectional area of hollow shaft,

$$A_2 = \frac{\pi}{4} \times (D^2 - d^2)$$

$$= \frac{\pi}{4} \times \left\{D^2 - (150 \times 10^{-3})^2\right\}$$

$$= \frac{\pi}{4}(D^2 - 0.0225)$$

Since the cross-sectional area of both the shafts are therefore equating A_1 and A_2,

$$\frac{\pi}{4}(200 \times 10^{-3})^2 = \frac{\pi}{4}(D^2 - 0.0225)$$

$$0.031 = D^2 - 0.0225$$

$$\Rightarrow D^2 = 0.031 + 0.0225 = 0.0535 \; m^2$$

$$D = 0.24$$

we also know that torque transmitted by the solid shaft,

$$T_1 = \frac{\pi}{16} \times \tau \times D_1^3$$

$$= \frac{\pi}{16} \times \tau \times (200 \times 10^{-3})^3$$

$$= 5 \times 10^{-4} \pi \tau \text{ N-m}$$

similarly, torque transmitted by the hollow shaft,

$$T_2 = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$$

$$= \frac{\pi}{16} \times \tau \times \left\{\frac{(0.25)^4 - (150 \times 10^{-3})^4}{0.25}\right\}$$

$$= 8.5 \times 10^{-4} \pi \tau \text{ N-m}$$

$$\therefore \frac{\text{Power transmitted by hollow shaft}}{\text{power transmitted by solid shaft}}$$

$$= \frac{T_2}{T_1} = \frac{8.5 \times 10^{-4} \pi \tau}{5 \times 10^{-4} \pi \tau} = 1.7$$

b) Ratio of angles of twist in both the shafts.

we know that relation for angle of twist for a shaft.

$$\frac{\tau}{R} = \frac{C.\theta}{l}$$

$$\Rightarrow \theta = \frac{\tau l}{RC}$$

Angel of twist for the solid shaft,

$$\theta_1 = \frac{\tau l}{RC} = \frac{\tau l}{0.1C} \quad \left(R = \frac{D_1}{2} = \frac{200 \times 10^{-3}}{2} = 0.1 m\right)$$

similarly angle of twist for the follow shaft,

$$\theta_2 = \frac{\tau l}{RC} = \frac{\tau l}{0.12C} \quad \left(R = \frac{D}{2} = \frac{0.24}{2} = 0.12\right)$$

$$\frac{\text{Angle of twist of hollow shaft}}{\text{Angle of twist of solid shaft}}$$

$$= \frac{\theta_2}{\theta_1} = \frac{\cancel{\tau l}/0.12\cancel{C}}{\cancel{\tau l}/0.1\cancel{C}} = \frac{0.1}{0.12} = 0.84$$

Ex - 27.15

A solid steel shaft of 60 mm diameter is to be replaced by a hollow steel shaft of the same material with internal diameter equal to half of the external diameter. Find the diameters of the hollow shaft and saving in material, if the maximum allowable shear stress is same for both shafts.

Given

$D = 60 mm = 60 \times 10^{-3} m$

Let $D = ?$

$d = ? \; (d = D_1/2)$

$\tau = ?$

We know that torque transmitted by the solid shaft,

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times \tau \times (60 \times 10^{-3})^3 \quad \ldots\ldots (i)$$

torque transmitted by the hollow shaft,

$$T_1 = \frac{\pi}{16} \times \tau \times \left(\frac{D_1^4 - d^4}{D_1}\right)$$

$$= \frac{\pi}{16} \times \tau \times \left\{\frac{D_1^4 - (0.5 D_1)^4}{D_1}\right\}$$

$$= \frac{\pi}{16} \times \tau \times 0.9375 \, D_1^3 \quad \ldots\ldots (ii)$$

Since the torque transmitted and allowable shear stress both the cases are same, therefore equating the equations (i) and (ii)

$$\frac{\pi}{16} \times \tau \times (60 \times 10^{-3})^3 = \frac{\pi}{16} \times \tau \times 0.9375 \, D_1^3$$

$$D_1^3 = \frac{(60 \times 10^{-3})^3}{0.9375} = 2.304 \times 10^{-4}$$

$D_1 = 1.33 \times 10^{-4}$

and $d = \dfrac{1.33 \times 10^{-4}}{2} = 6.65 \times 10^{-5}$

saving in material

we know that saving in material

$$= \frac{\left\{\cancel{\frac{\pi}{4}}(60 \times 10^{-3})^2\right\} - \cancel{\frac{\pi}{4}}\left\{(1.33 \times 10^{-4})^2 - (6.65 \times 10^{-5})\right\}}{\cancel{\frac{\pi}{4}}(60 \times 10^{-3})^2}$$

$$= \frac{\cancel{(2.83 \times 10^{-3}) + (5.23 \times 10^{-5})}}{\cancel{2.83 \times 10^{-3}}}$$

$$= \frac{(3.6 \times 10^{-3}) + (6.65 \times 10^{-5})}{3.6 \times 10^{-3}} = 1.019$$

Ex - 27.16

A solid shaft of 80mm diameter is to be replaced by a hollow shaft of external diameter 100mm. Determine the internal diameter of the hollow shaft if the same power is to be transmitted by both the shaft at the same angular velocity and shear stress.

Given.

$D = 80mm = 80 \times 10^{-3} m$

$(D_1) = 100mm = 100 \times 10^{-3}$

$d = ?$

$\tau = ?$

we know that torque transmitted by the solid shaft,

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times \tau \times (80 \times 10^{-3})^3 \quad \ldots (i)$$

$=$

Torque transmitted by the hollow shaft,

$$T_1 = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$$

$$= \frac{\pi}{16} \times \tau \times \left\{\frac{(100 \times 10^{-3})^4 - d^4}{100 \times 10^{-3}}\right\} \quad \cdots (ii)$$

Since both the torques are equal, therefore equating equations (i) and (ii).

$$\cancel{\frac{\pi}{16}} \times \cancel{\tau} \times (80 \times 10^{-3})^3 = \cancel{\frac{\pi}{16}} \times \cancel{\tau} \times \left\{\frac{(100 \times 10^{-3})^4 - d^4}{100 \times 10^{-3}}\right\}$$

$$(80 \times 10^{-3})^3 = \frac{(100 \times 10^{-3})^4 - d^4}{\cancel{100 \times 10^{-3}}} = (100 \times 10^{-3})^3 -$$

$$\frac{d^4}{100 \times 10^{-3}} = (100 \times 10^{-3})^3 - (80 \times 10^{-3})^3 = 4.88 \times 10^{-4}$$

$$\Rightarrow d^4 = (4.88 \times 10^{-4}) \times (100 \times 10^{-3}) = 4.88 \times 10^{-5}$$

$$d = 1.49 \times 10^{-5}$$

Ex - 27.17

A solid aluminium shaft 1m long and of 50mm diameter is to be replaced by a hollow shaft of the s length and same outside diameter, so that the hollow shaft could carry the same torque and has the same angle of twist. What must be the inner diameter of the hollow shaft?

Take modulus of rigidity for the aluminium as 28 GPa and that for steel as 85 GPa, Given

$l_A = 1m$

$D_A = 50mm = 50 \times 10^{-3} m$

$l_s = 1m$

$D_s = 50mm = 50 \times 10^{-3} m$

$C_A = 28\ GPa = 28 \times 10^9\ N/m^2$

$C_s = 85\ GPa = 85 \times 10^9\ N/m^2$

$d_s = ?$

we know that polar moment of inertia of the solid aluminium shaft,

$$J_A = \frac{\pi}{32} \times D^4$$

$$= \frac{\pi}{32} \times (50 \times 10^{-3})^4$$

we also know that relation for angle of twist

$$\frac{T}{J} = \frac{C \cdot \theta}{l}$$

$$\theta = \frac{T \cdot l}{JC}$$

$$\therefore \theta_A = \frac{T_A \cdot l_A}{\frac{\pi}{32} \times (50 \times 10^{-3})^4 \times 28 \times 10^9} \text{ rad.}$$

$$\theta_s = \frac{T_s \cdot l_s}{\frac{\pi}{32} \times \{(50 \times 10^{-3})^4 - (d)^4\} \times 85 \times 10^9} \text{ rad.}$$

since both the angle of twists (i.e θ_A and θ_B) are same, there force equating these values,

$$\frac{T_A \cdot l_A}{\cancel{\frac{\pi}{32}} \times (50 \times 10^{-3})^4 \times 28 \times \cancel{10^9}} = \frac{T_s \cdot l_s}{\cancel{\frac{\pi}{32}} \times \{(50 \times 10^{-3})^4 - (d)^4\} \times 85 \times \cancel{10^9}}$$

Substituting $T_A = T_S$

$$(50\times10^{-3})^4\times28 = \{(50\times10^{-3})^4-(d)^4\}\times85$$

$$1.75\times10^{-4} = (6.25\times10^{-6}) - 85d^4$$

$$85d^4 = (6.25\times10^{-6}) - (1.75\times10^{-4}) = -1.69\times10^{-}$$

$$d^4 = \frac{1.69\times10^{-4}}{85} = 1.989\times10^{-6}$$

$$d = 1.188\times10^{-6}$$

Ex-27.18

A hollow steel shaft shaft of 300mm extrnal diam and 200mm internal diameter has to be replaced by a so alloy shaft. Assuming the same values of polar modulus for calculate the diameter of the latter and work out the ra of their torsional rigidities. Take C for steel as 2.4C for alloy.

Given

$D = 300mm = 300\times10^{-3}m$

$d = 200 mm = 200\times10^{-3}m$

$C_s = 2.4$

Diameter of the solid alloy shaft.

Let $D_1 = ?$

we know that polar modulus of hollow steel shaft.

$$Z_s = \frac{\pi}{16D}(D^4-d^4)$$

$$= \frac{\pi}{16\times300\times10^{-3}}\{(300\times10^{-3})^4-(200\times10^{-3})^4\}$$

$=$