



GOVT. POLYTECHNIC KANDHAMAL , PHULBANI

(State Council for Technical Education and Vocational Training , Odisha)

Th-1 Structural Mechanics

3rd Semester , Diploma

Lecture Notes

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Review of Basic Concepts

→ Defination of Centre of gravity! → It is a point may be found out in a body throo which the resultant of such Parallel force act this point throo which the hole weight of the body acts irrespect of it position is known as centre of gravity.

→ It may be noted that every body has one & only one centre of gravity.

^{2 mark} → Centroid! → A plane figure likes triangle, quadric lateral, circle etc have only area but no mass. The centre of area such figures is known as centroid.

→ Methods of centre of gravity! → The centre of gravity may be found out by any one of the following 2 methods

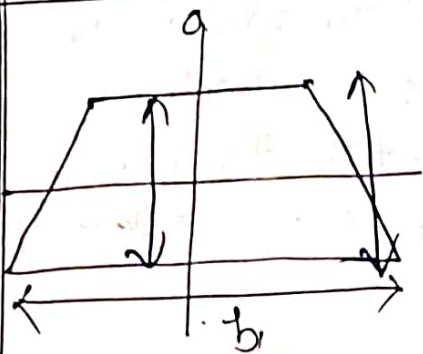
1. By Geometrical consideration.
2. By moments
3. By Graphical method.

We shall discuss the procedure for finding out the C.G of simple figure by Geometrical consideration. & By moment one by one.

→ Centre of gravity by Geometrical methods! →

	Figure	C.G	Area	Volume
1.		$\bar{x} = l/2$ $\bar{y} = h/2$	$l \times h$	$l \times h \times \text{height}$
2.		$\bar{x} = b$ $\bar{y} = h/3$	$\frac{1}{2} \times b \times h$	$\frac{1}{2} \times b \times h \times l$

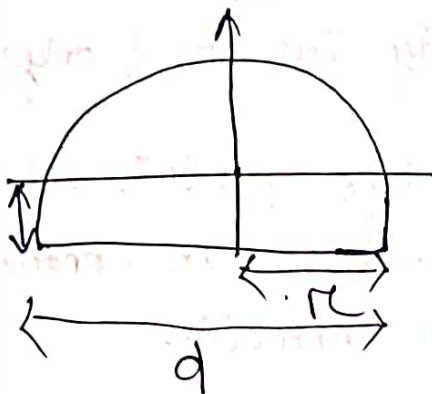
3.



$$\bar{x} = 0 \quad \left(\frac{b+2a}{b+a} \right) \left(\frac{a+b}{2} \right) h \quad \left(\frac{a+b}{2} \right) h^2$$

$$\bar{y} = h/3$$

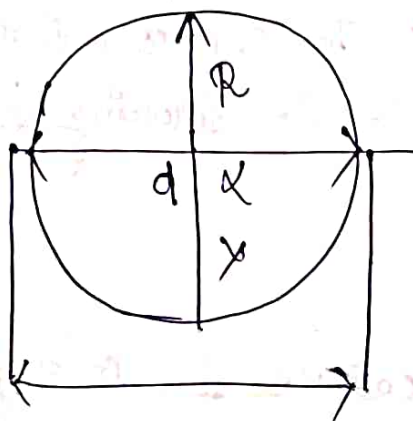
4.



$$\bar{x} = 0 \quad \frac{\pi R^2}{2} \quad \frac{\pi R^2}{2} \times d$$

$$\bar{y} = \frac{4R}{3\pi}$$

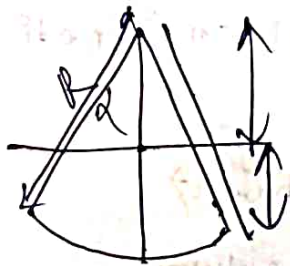
5.



$$\bar{x} = d/2 = R \quad \pi R^2 \quad \pi R^2 L$$

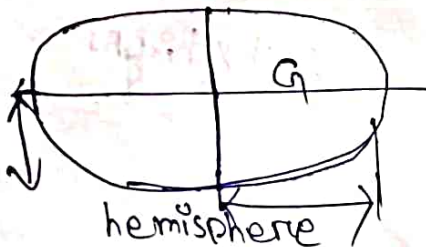
$$\bar{y} = \frac{d}{2} \times R$$

6.



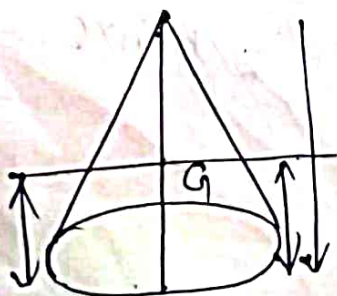
$$\frac{2R}{3} \frac{\sin \alpha}{\alpha} \bar{y}$$

7.



$$CG \cdot \bar{y} = \frac{3\pi}{8}$$

8.



$$\bar{y} = h/4$$

→ Centre of gravity of plane figures:

The plane geometrically figures have only area but no mass.

Let $\bar{x}$ & $\bar{y}$ be the co-ordinates of the CG with respect to some axis of reference i.e.

$$\text{then } \bar{x} = \frac{a_1x_1 + a_2x_2 + \dots}{a_1 + a_2 + \dots}$$

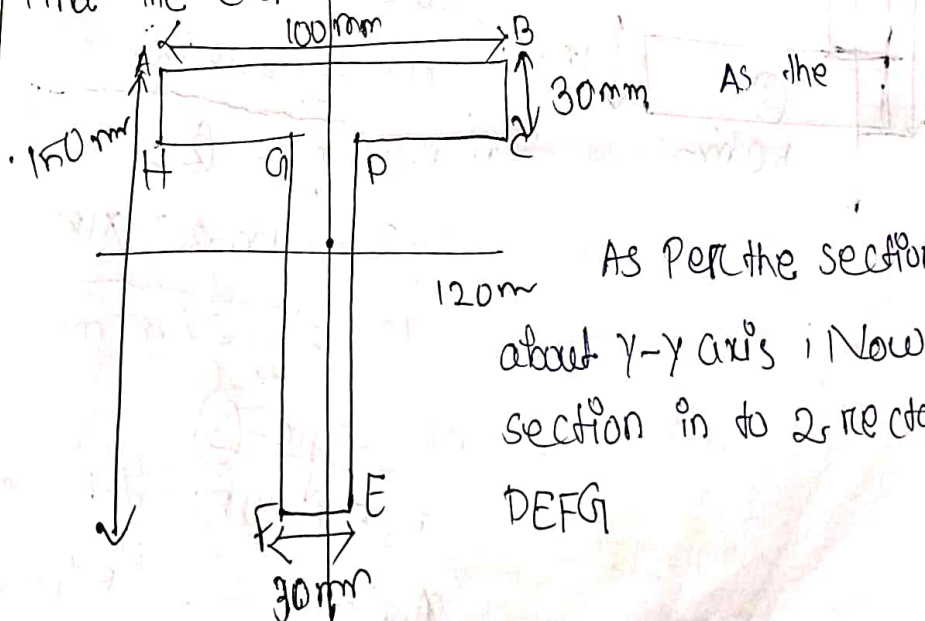
$$\bar{y} = \frac{a_1y_1 + a_2y_2 + \dots}{a_1 + a_2 + \dots}$$

where $a_1, a_2, a_3, \dots$ are the area in to which the whole figure is divided. $x_1, x_2, x_3, \dots$ are the x -co-ordinates of the areas $a_1, a_2, a_3, \dots$ on x -axis with respect to some axis of reference.

$y_1, y_2, \dots$ are the respective co-ordinates of the area $a_1, a_2, \dots$ on y -axis with respect to some axis of reference.

CG of some symmetrical section:→

1. Find the CG of a 100mm, 150mm, 30mm T-section?



As Per the section is symmetrical about y -axis. Now split up the section in to 2 rectangles, ABCH & DEFG.

→ In rectangle ABCD
 $a_1 = 100 \times 300 = 30000 \text{ mm}^2$
 $y_1 = \left(150 - \frac{30}{2}\right) = 135 \text{ mm}$

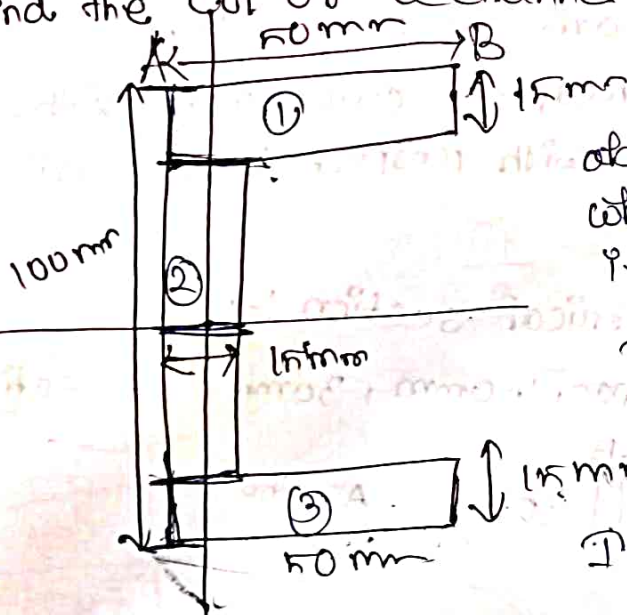
→ In rectangle DEFG
 $a_2 = 120 \times 30 = 3600 \text{ mm}^2$
 $y_2 = \frac{120}{2} = 60 \text{ mm}$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(30000 \times 135) + (3600 \times 60)}{30000 + 3600}$$

$$= 94.1 \text{ mm}$$

2. Find the CG of a channel section $100 \text{ mm} \times 50 \text{ mm} \times 15 \text{ mm}$



As the section is symmetrically about x-x axis, Now split up the whole section into 3 rectangular in rectangular -

In rectangle - ①

$$a_1 = 50 \times 15 = 750 \text{ mm}^2$$

$$x_1 = \frac{50}{2} = 25 \text{ mm}$$

In rectangle - ②

$$a_2 = (100 - 30) \times 15 = 1050 \text{ mm}^2$$

$$x_2 = \frac{15}{2} = 7.5 \text{ mm}$$

In rectangle - ③

$$a_3 = 50 \times 15 = 750 \text{ mm}^2$$

$$x_3 = \frac{50}{2} = 25 \text{ mm}$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(750 \times 25) + (1050 \times 75) + (750 \times 25)}{750 + 1050 + 750}$$

$$= 17.8 \text{ mm}$$

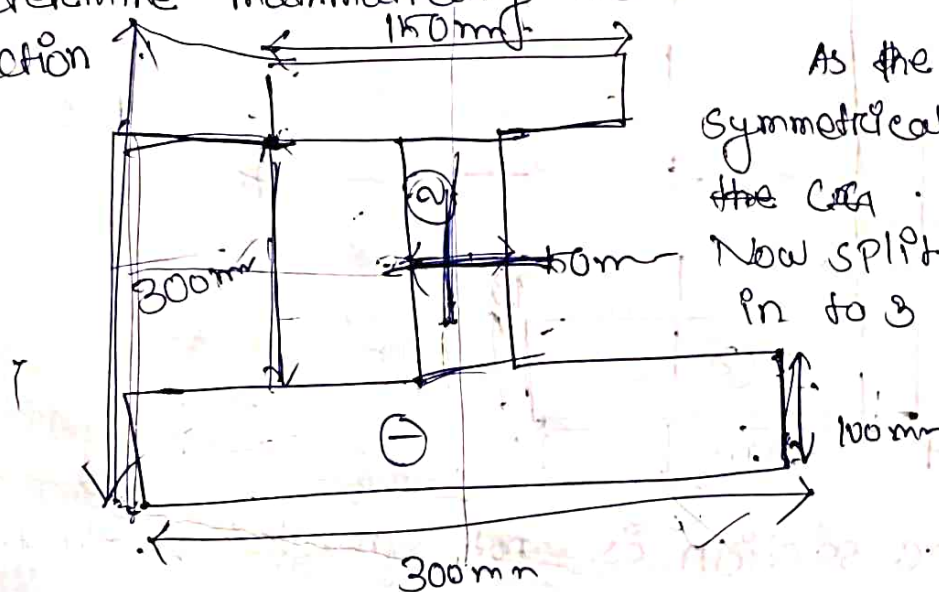
Q3 An I-section has the following dimensions in mm.

Bottom flange = 300×100

Top flange = 150×50

web = 300×50

Determine mathematically the position of CG of the section



As the section is symmetrically about y-axis, the CG is

Now split up the section into 3 rectangles

1. Bottom flange

$$a_1 = 300 \times 100 = 30,000 \text{ mm}^2$$

$$y_1 = \frac{100}{2} = 50 \text{ mm}$$

web

$$2. a_2 = 300 \times 50 = 15,000 \text{ mm}^2$$

$$y_2 = 100 + \frac{300}{2} = 250 \text{ mm}$$

3. Top

$$a_3 = 150 \times 50 = 7,500 \text{ mm}^2$$

$$y_3 = 100 + 300 + \frac{50}{2} = 412.5 \text{ mm}$$

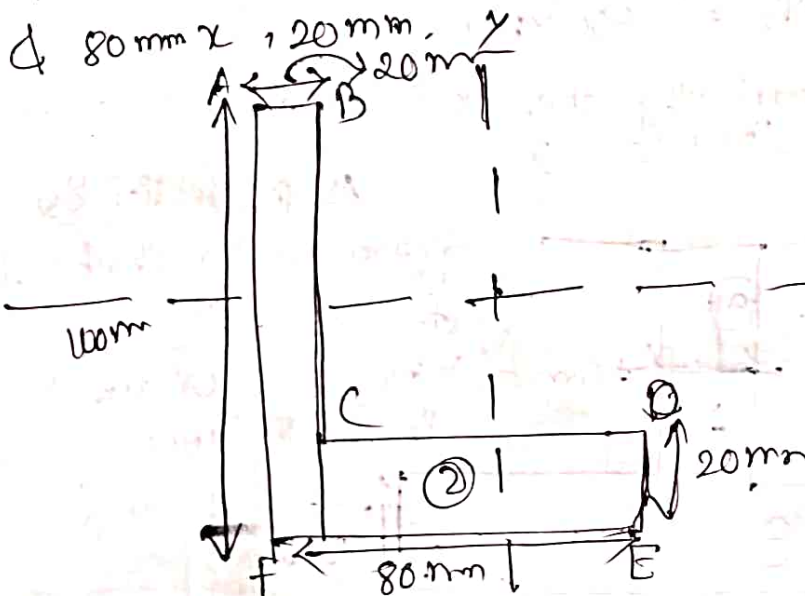
$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(30000 \times 70) + (15000 \times 250) + (7500 \times 425)}{30000 + 15000 + 7500}$$

$$= 160.7 \text{ mm}$$

Centre of gravity of unsymmetrical section! →
Find the C.G. of an unequal angle section 100mm

①



As the section is not symmetrical about any axis, we have to find out the values of $\bar{x}$ & $\bar{y}$ for the angle section.

Now split up the section into 2 rectangles

rectangle ① $a_1 = 100 \times 20 = 2000 \text{ mm}^2$

$$y_1 = \frac{100}{2} = 50 \text{ mm}$$

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

In rectangle-2

$$a_2 = (80 \cdot 20) \times 20 = 1200 \text{ mm}^2$$

$$x_2 = 20 + \frac{60}{2} = 50 \text{ mm}$$

$$y_2 = \frac{20}{2} = 10 \text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{2000 \times 10 + 1200 \times 50}{2000 + 1200}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{2000 \times 50 + 1200 \times 10}{2000 + 1200}$$

$$= 35 \text{ mm} \quad \underline{\text{Ans}}$$

Moment of Inertia

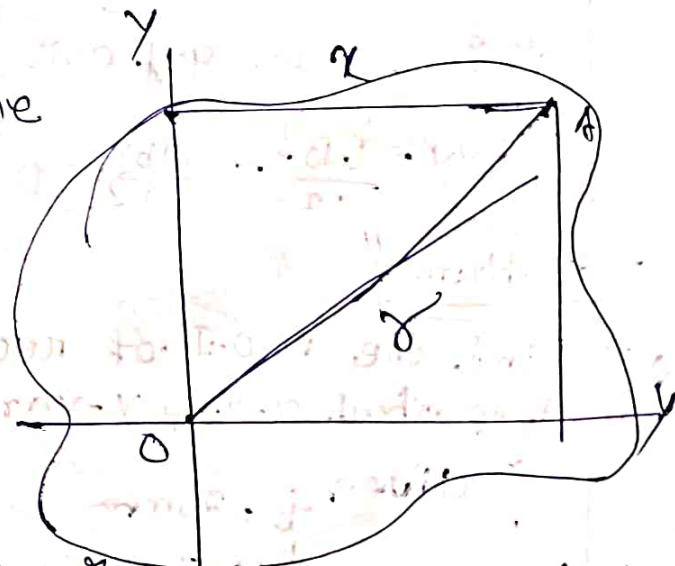
→ The moment of Inertia of a plane figure with respect to x & y axis in its plane are expressed

$$\text{as } I_{xx} = \int y^2 dA$$

$$I_{yy} = \int x^2 dA$$

→ I_{xx} & I_{yy} are also known as second moment of area about the axes as it's distance is separated from corresponding axis.

Unit → Unit of moment of Inertia of area as expressed as m^4 & mm^4 .



→ Method of moment of inertia!→

The moment of Inertia of a plane area may be found by any one of the following.

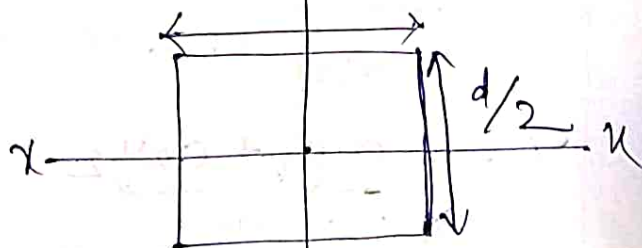
- By Routh's Rule
- By Integration
- Moment of Inertia of a plane figure!→

1) Rectangle section!→

The M.O.I of about x-axis is $I_{xx} = \frac{bd^3}{12}$

Similarly in y-y axis as

$$I_{yy} = \frac{db^3}{12}$$

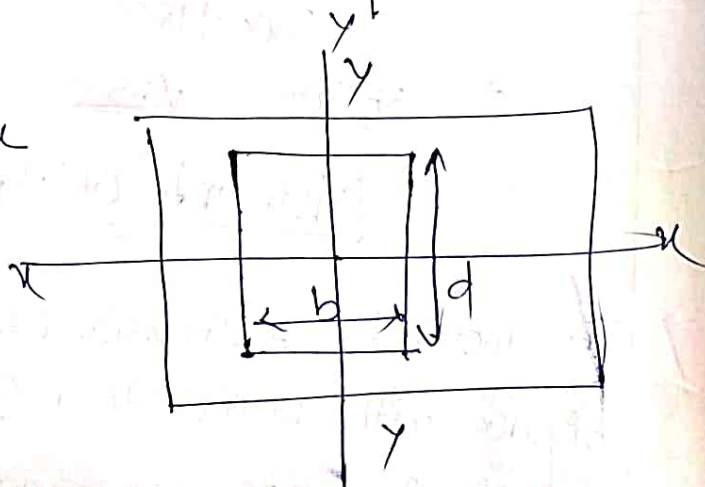


2) Hollow Rectangle section!→

The M.O.I of about x-x axis is $I_{xx} = \frac{Bd^3}{12} - \frac{bd^3}{12}$

Similarly on y-y axis

$$I_{yy} = \frac{Bb^3}{12} - \frac{db^3}{12}$$



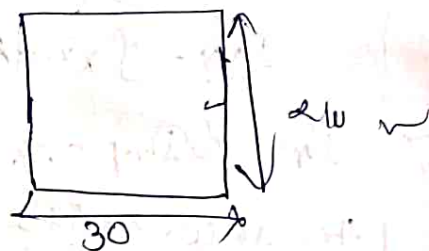
Problem-1

- Find the M.O.I of rectangle section 30mm wide & 40mm deep about x-x & y-y axis.

Given $b = 30\text{mm}$
 $d = 40\text{mm}$

$$I_{xx} = \frac{bd^3}{12} = \frac{30 \times 40^3}{12} = 160 \times 10^3 \text{ mm}^4$$

$$I_{yy} = \frac{db^3}{12} = \frac{40 \times 30^3}{12} = 90 \times 10^3 \text{ mm}^4$$



2. Find the M.O.I of a hollow rectangular section about its centre of gravity if the external dimensions are breadth 60mm depth 80mm & Internal dimensions are breadth 30mm & depth 40mm resp.

Given

$$B = 60\text{mm}$$

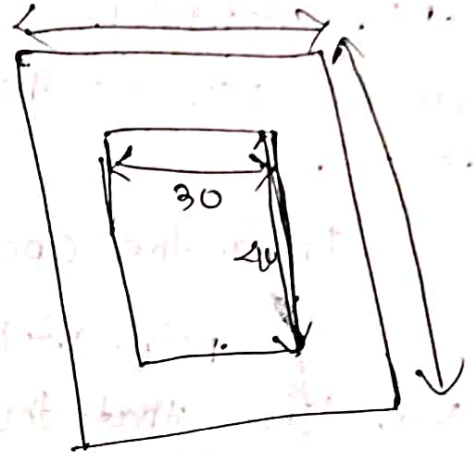
$$D = 80\text{mm}$$

$$b = 30\text{mm}$$

$$d = 40\text{mm}$$

$$\begin{aligned} I_{xx} &= \frac{BD^3}{12} - \frac{db^3}{12} \\ &= \frac{60 \times 80^3}{12} - \frac{30 \times 40^3}{12} \\ &= 24000 \times 10^3 \text{mm}^4 \end{aligned}$$

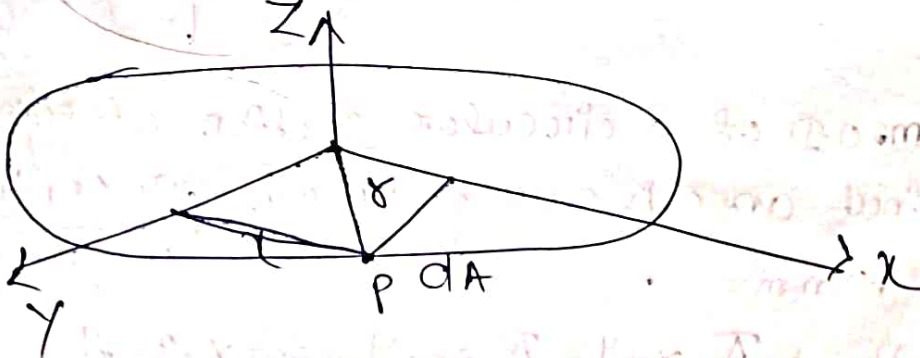
$$\begin{aligned} I_{yy} &= \frac{DB^3}{12} - \frac{db^3}{12} \\ &= \frac{80 \times 60^3}{12} - \frac{40 \times 30^3}{12} \\ &= 13500 \times 10^3 \text{mm}^4 \end{aligned}$$



→ Theorem of Perpendicular axis →

It states, If I_{xx} & I_{yy} be the M.O.I of a plane section about 2 $\perp$ axes meeting at 'O' the M.O.I I_{zz} about the axis Z-Z, $\perp$ to the plane & passing through the intersection of x-x & y-y is given by

$$I_{zz} = I_{xx} + I_{yy}$$



→ Proof: → Consider a small lamina (P) of area dA having co-ordinates as x & y along of x & y 2 mutually $\perp$ axes on a plane section.

Now consider a plane $oz \perp$ to ox & oy let, (x) be the distance of the coming (P) from $z-z$ axis such that $OP = r$.

From the Geometric of the figure

$$r^2 = x^2 + y^2$$

We know that the M.O.I of the lamina (P) about $x-x$ axis,

$$I_{xx} = dA y^2 / y^2 dA$$

$$I_{yy} = dA x^2 / x^2 dA$$

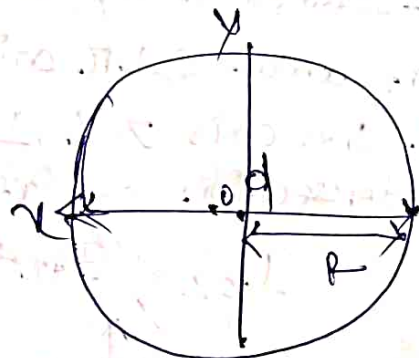
$$I_{zz} = dA r^2 = dA (x^2 + y^2) \\ = dA x^2 + dA y^2$$

$$I_{zz} = I_{xx} + I_{yy}$$

→ Moment of inertia of a circular section →

The moment of inertia of about $x-x$ axis is equal to $y-y$ axis is.

$$I_{yy} = I_{xx} = \frac{\pi}{64} \times d^4$$



→ Problem-73

Find the M.O.I of a circular section of 30mm diameter about an axis passing through its centre

Given, $d = 30 \text{ mm}$

$$I_{xx} = I_{yy} = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times 30^4 = 302 \times 10^3 \text{ mm}^4$$

→ Moment of Inertia of a hollow circular section

→ Here also the M.O.I of about x-axis equal to y-axis $I_{xx} = I_{yy} = \frac{\pi}{64} (D^4 - d^4)$

→ Problem

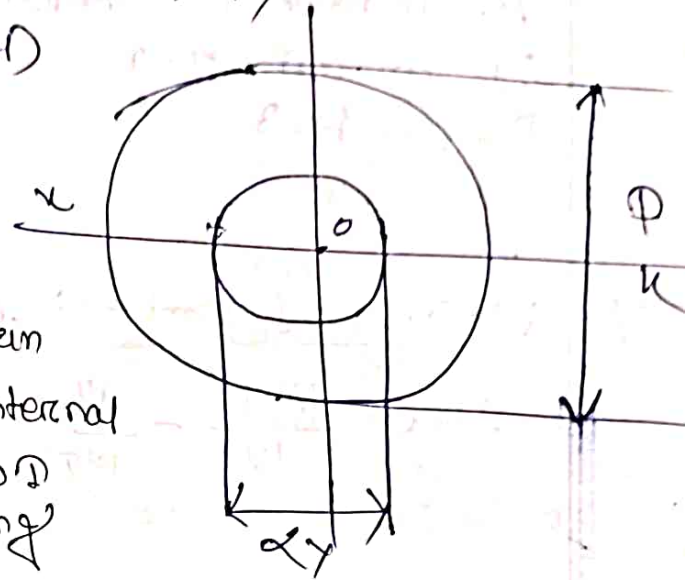
A hollow circular section has an external diameter of 80 mm & Internal diameter of 60 mm find the M.O.I about the horizontal axis passing through its centre

Given $D = 80 \text{ mm}$

$d = 60 \text{ mm}$

$$I_{xx} = \frac{\pi}{64} (D^4 - d^4)$$

$$= \frac{\pi}{64} (80^4 - 60^4) = 137.4 \times 10^3 \text{ mm}^4$$



→ Theorem of parallel axis

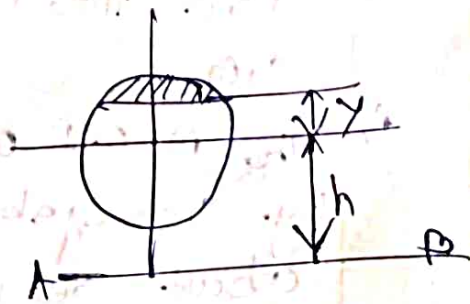
It states, If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I_G , then moment of Inertia of the area about any other axis AB parallel to the 1st & at a distance h from the centre of gravity is given by $I_{AB} = I_G + Ah^2$

area about an axis AB.

I_G = M.O.I of the area about its C.G

A = Area of the section.

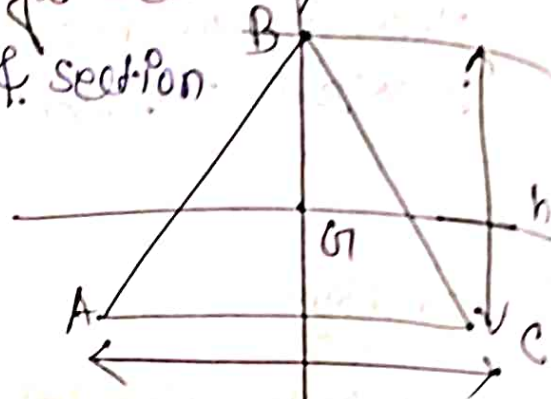
h = Distance betⁿ C.G of the section & axis AB



→ Moment of Inertia of a triangle section! →

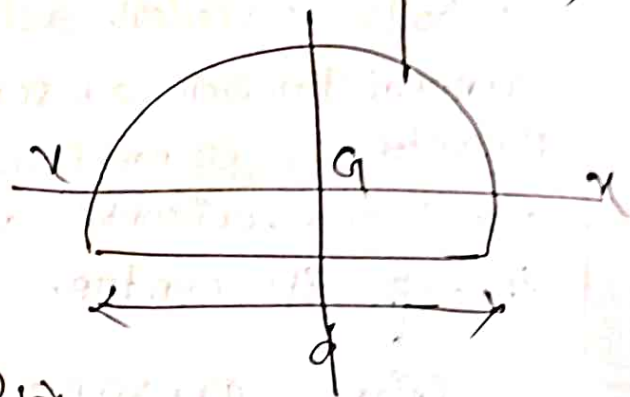
The moment of inertia of the $\triangle$ section of an axis $x-x$

$$I_{xx} = \frac{bh^3}{36}$$



→ M.O.I of semi-circle

$$I_{xx} = \frac{\pi d^4}{128} - \frac{d^4}{18\pi}$$



→ M.O.I of Composite section! →

→ The M.O.I of a composite section may be found out by the following set steps

1/ First of all, split up the given section into plane area.

2/ Find the M.O.I of these M.O.I about required axis (AB) by the theorem of parallel axis.

$$i.e. I_{AB} = I_G + Ah^2$$

3/ Now, transfer the M.O.I about required axis (AB) by the theorem of parallel axis i.e. $I_{AB} = I_G + Ah^2$

2/ Find the M.O.I of these area of their respective C.G.

→ The M.O.I of the given section may now be obtained by the algebraic sum of the moment of inertia about the required axis.

⇒ Problem 17

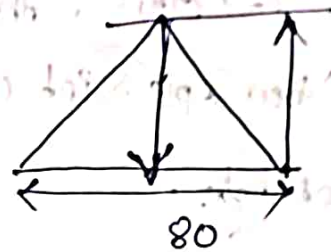
Find the M.O.I of a triangle section ABC has base 80 mm & height 60 mm respectively.

Given $b = 80 \text{ mm}$

height (h) = 60 mm

$$I_{xx} = \frac{bh^3}{12} = \frac{80 \times 60^3}{12}$$

$$= 1440000 \text{ mm}^4$$



⇒ compute the moment of inertia of the above area about axis k-k.

→ we know that M.O.I of section

① about its C.G. & llr to axis k-k

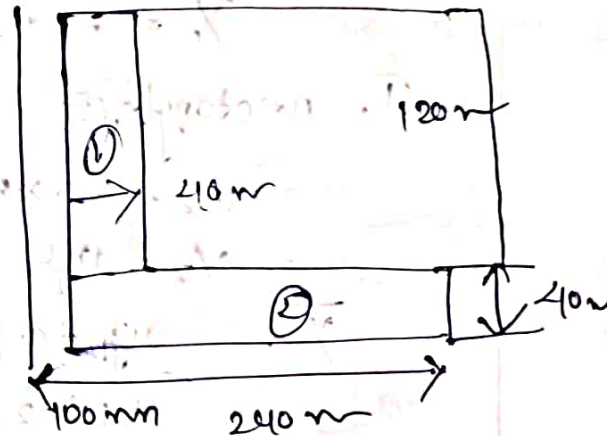
$$I_{01} = \frac{bd^3}{12} = \frac{120 \times 40^3}{12} = 640 \times 10^3 \text{ mm}^4$$

$$h_1 = 100 + \frac{40}{2} = 120 \text{ mm}$$

$$\text{M.O.I, } I_{KK1} = I_{01} + \alpha h_1^2$$

$$= 640 \times 10^3 + [(120 \times 40) \times 120^2]$$

$$= 69.76 \times 10^6 \text{ mm}^4$$



Similarly The M.O.I of section ② about the C.G. & llr to axis k-k.

$$I_{02} = \frac{40 \times 240^3}{12} = 46.08 \times 10^6 \text{ mm}^4$$

$$h_2 = 100 + \frac{240}{2} = 220 \text{ mm}$$

$$\therefore \text{M.O.I, } I_{KK} = I_{02} + \alpha_2 h_2^2$$

$$= 46.08 \times 10^6 + [(240 \times 40) \times 220^2]$$

$$= 510.72 \times 10^6 \text{ mm}^4$$

∴ Now the M.O.I of the whole area about axis k-k

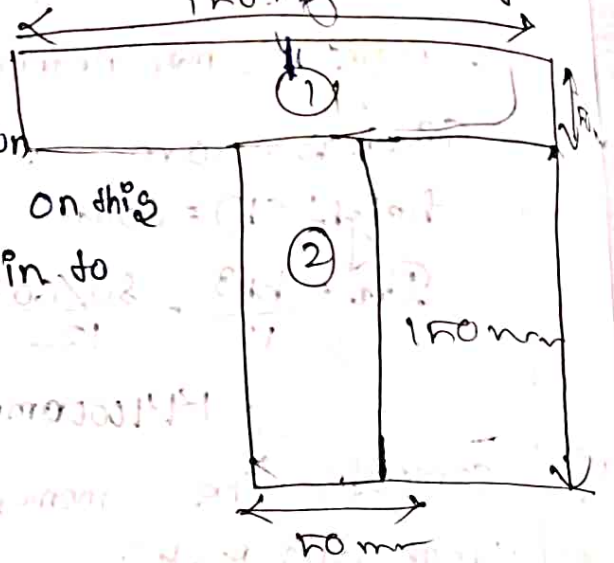
$$\therefore I_{KK} = I_{KK1} + I_{KK2}$$

$$= 69.76 \times 10^6 + 510.72 \times 10^6$$

$$= 580.48 \times 10^6 \text{ mm}^4 \quad \text{Ans}$$

6) Find the M.O.I of a T-section with flange as $150\text{mm} \times 50\text{mm}$ & web as $150\text{mm} \times 50\text{mm}$ about x-x axis & y-y axis through the C.G. of the section.

→ This T-section is symmetrical section about y-y axis, therefore C.G. lies on this axis. Now split up the section into 2 rectangles.



In rectangle ①

$$a_1 = 150 \times 50 = 7500 \text{ mm}^2$$

$$y_1 = 50 + \frac{50}{2} = 75 \text{ mm}$$

In rectangle ②

$$a_2 = 50 \times 150 = 7500 \text{ mm}^2$$

$$y_2 = \frac{150}{2} = 75 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 75) + (7500 \times 75)}{7500 + 7500} = 125 \text{ mm}$$

M.O.I of about x-x axis

In rectangle - ①

$$I_{a1} = \frac{150 \times 50^3}{12} = 1.5625 \times 10^8 \text{ mm}^4$$

$$h_1 = 175 - 125 = 50 \text{ mm}$$

$$I_{x1} = I_{a1} + a_1 h_1^2$$

$$= (1.5625 \times 10^8) + (7500 \times 50^2)$$

$$= 20.3125 \times 10^8 \text{ mm}^4$$

In rectangle - ②

$$I_{a2} = \frac{50 \times 150^3}{12} = 14.0625 \times 10^8 \text{ mm}^4$$

$$h_2 = 125 - 75 = 50 \text{ mm}$$

$$I_{x2} = I_{a2} + a_2 h_2^2$$

$$= 14.0625 \times 10^8 + (7500 \times 50^2)$$

$$= 32.8125 \times 10^8 \text{ mm}^4$$

Now, $I_{xx} = I_{xx1} + I_{xx2}$

$$= 20.312 \times 10^6 + 32.8125 \times 10^6$$

$$= 53.125 \times 10^6 \text{ mm}^4$$

M.O.I about $y-y$ axis in rectangle ①

$$\frac{db^3}{12} = \frac{50 \times 150^3}{12} = 12.0625 \times 10^6 \text{ mm}^4$$

In rectangle ②

$$\frac{db^3}{12} = \frac{150 \times 50^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

$$\therefore I_{yy} = 140.6625 \times 10^6 + 1.5625 \times 10^6$$

$$= 142.225 \times 10^6 \text{ mm}^4$$

7. An I-section is made up of 3 rectangles find the M.O.I of the section about the horizontal axis passing through the C.G. of the section.

This I-section is symmetrical about $y-y$ axis, therefore the C.G. lies on this axis.

Now split on the section in to 3 rectangles

In rectangle ①

$$a_1 = 60 \times 20 = 1200 \text{ mm}^2$$

$$y_1 = 20 + 100 + \frac{20}{2} = 130 \text{ mm}$$

In rectangle ②

$$a_2 = 100 \times 20 = 2000 \text{ mm}^2$$

$$y_2 = 20 + \frac{100}{2} = 70 \text{ mm}$$

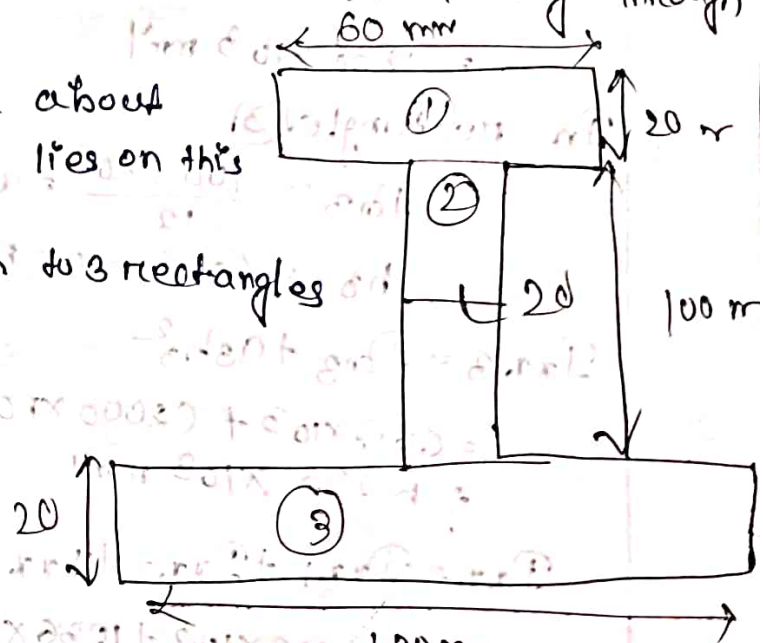
In rectangle ③

$$a_3 = 100 \times 20 = 2000 \text{ mm}^2$$

$$y_3 = \frac{20}{2} = 10 \text{ mm}$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000}$$

$$= 60.8 \text{ mm}$$



The M.O.I of the section about x-x axis is

In rectangle ①

$$I_{01} = \frac{60 \times 20^3}{12} = 40 \times 10^3 \text{ mm}^4$$

$$h_1 = 130 - 60.8 = 69.2 \text{ mm}$$

$$I_{xx1} = I_{01} + a_1 h_1^2$$

$$= 40 \times 10^3 + (1200 \times 69.2^2)$$

$$= 5786 \times 10^3 \text{ mm}^4$$

In rectangle ②

$$I_{02} = \frac{20 \times 100^3}{12} = 16.666 \times 10^3 \text{ mm}^4$$

$$h_2 = 70 - 60.8 = 9.2 \text{ mm}$$

$$I_{xx2} = I_{02} + a_2 h_2^2$$

$$= 16666.6 \times 10^3 + (2000 \times 9.2^2)$$

$$= 1836 \times 10^3 \text{ mm}^4$$

In rectangle ③

$$I_{03} = \frac{100 \times 20^3}{12} = 66.7 \times 10^3 \text{ mm}^4$$

$$h_3 = 60.8 + 10 = 70.8 \text{ mm}$$

$$I_{xx3} = I_{03} + a_3 h_3^2$$

$$= 66.7 \times 10^3 + (2000 \times 70.8^2)$$

$$= 6228 \times 10^3 \text{ mm}^4$$

$$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}$$

$$= 5786 \times 10^3 + 1836 \times 10^3 + 6228 \times 10^3$$

$$= 12850 \times 10^3 \text{ mm}^4$$

8) Find the M.O.I about the centroidal x-x & y-y axes of the angle section.

7) The angle section is not symmetrical about both axes. Now split up the section into 2 rectangles:

→

In rectangle ①

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

$$y_1 = \frac{100}{2} = 50 \text{ mm}$$

In rectangle ②

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

$$x_2 = 20 + \frac{60}{2} = 50 \text{ mm}$$

$$y_2 = \frac{20}{2} = 10 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200}$$

$$= 35 \text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm}$$

M.O.I about C.G in x-x axis

In rectangle ① $I_{xx} = \frac{20 \times 100^3}{12} = 1.667 \times 10^6 \text{ mm}^4$

$$h_1 = 50 - 35 = 15 \text{ mm}$$

$$\begin{aligned} I_{xx1} &= I_{xx} + a_1 h_1^2 \\ &= 1.667 \times 10^6 + (2000 \times 15^2) \\ &= 2.117 \times 10^6 \text{ mm}^4 \end{aligned}$$

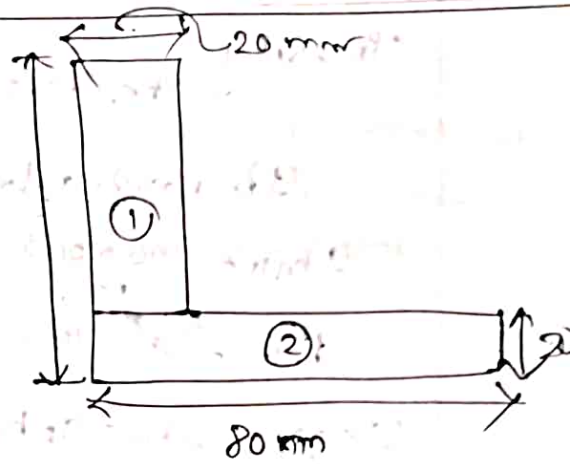
In rectangle ② $I_{xx} = \frac{60 \times 20^3}{12} = 0.04 \times 10^6 \text{ mm}^4$

$$h_2 = 35 - 10 = 25 \text{ mm}$$

$$\begin{aligned} I_{xx2} &= I_{xx} + a_2 h_2^2 \\ &= 0.04 \times 10^6 + (1200 \times 25^2) \\ &= 0.74 \times 10^6 \text{ mm}^4 \end{aligned}$$

Total M.O.I of the whole section about x-x axis

$$\begin{aligned} I_{xx} &= I_{xx1} + I_{xx2} \\ &= 2.117 \times 10^6 + 0.74 \times 10^6 = 2.907 \times 10^6 \text{ mm}^4 \end{aligned}$$



Similarly The M.O.I through the C.G. & lie to $Y-Y$ axis —

In rectangle ①

$$I_{xx1} = \frac{100 \times 200^3}{12} = 0.067 \times 10^8 \text{ mm}^4$$

$$h_{11} = 25 - 10 = 15 \text{ mm}$$

$$\begin{aligned} I_{yy1} &= I_{xx1} + a_1 h_{11}^2 \\ &= 0.067 \times 10^8 + (2000 \times 15^2) \\ &= 0.517 \times 10^8 \text{ mm}^4 \end{aligned}$$

In rectangle ②

$$I_{xx2} = \frac{20 \times 60^3}{12} = 0.36 \times 10^8 \text{ mm}^4$$

$$h_{22} = 50 - 25 = 25 \text{ mm}$$

$$\begin{aligned} I_{yy2} &= I_{xx2} + a_2 h_{22}^2 \\ &= 0.36 \times 10^8 + (1200 \times 25^2) \\ &= 1.11 \times 10^8 \text{ mm}^4 \end{aligned}$$

Total M.O.I about an axis $Y-Y$ —

$$\begin{aligned} I_{yy} &= I_{yy1} + I_{yy2} \\ &= 0.517 \times 10^8 + 1.11 \times 10^8 \\ &= 1.627 \times 10^8 \text{ mm}^4, \text{ Ans} \end{aligned}$$

Rupinder

Simple stresses & strain

→ Stress → every material is elastic in nature. whenever some external system of forces act on a body it undergoes deformation. The molecules of the body set up some resistances to deformation. This resistance force unit area to deform is known as stress.

The stress is denoted by sigma (σ)

mathematically $\sigma = \frac{P}{A}$

where σ is stress

P load or force acting on the body.

A = crosssectional area of the body. in SI unit of force is N/mm² or KN/m²

→ Unit of stress is Pascal (Pa)

$$1 \text{ Pa} = 1 \text{ N/mm}^2$$

→ Strain → whenever a single force or system of forces acts on a body, it undergoes deformation. This deformation per unit length is known as strain.

OR
→ It is the ratio between change of length of the body to the original length of the body, is known as strain.

mathematically it is denoted by 'e' or $\epsilon = \frac{\Delta l}{l}$

where ϵ = strain of the body

Δl = change of length of the body

l = original length of the body

→ It is unitless

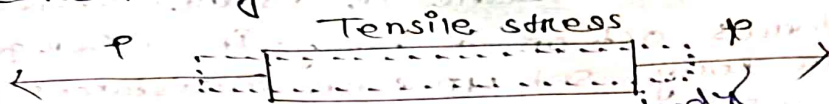
→ Types of stress → Stress is different types:

1. tensile stress

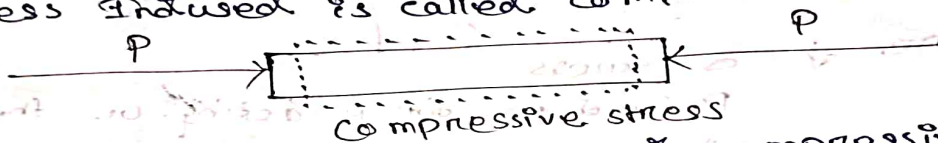
2. Compressive stress.

→ Tensile stress → when a section is subjected to two equal and opposite pull and the body tends to increase its length. the stress induced is called tensile stress.

The corresponding strain is called tensile strain.



compressive stress $\Rightarrow$ when a body is subjected to two equal and opposite push and the body tends to decrease its length the stress induced is called compressive stress.



The corresponding strain is called compressive strain.

Hooke's law $\Rightarrow$ It states, when a material is loaded with in its elastic limit the stress is proportional to the strain.

Mathematically, stress $\propto$ strain

$$\Rightarrow \sigma \propto e$$

$$\Rightarrow \frac{\sigma}{e} = E$$

$$\Rightarrow \frac{\sigma}{e} = E$$

$$\text{or } E = \frac{\text{stress}}{\text{strain}}$$

where

E is a constant of proportionality is known as young's modulus or modulus of elasticity.

σ = stress

e = strain

Deformation of a body due to force acting on it $\Rightarrow$

consider a body subjected to tensile stress. we know stress $e = \frac{P(\sigma)}{A}$ and strain $e = \frac{\Delta l}{l}$ and young's modulus

where Δl = Deformation of the body

P = load or force acting on the body.

l = length of the body

A = cross-sectional area of the body

E = Modulus of Elasticity for the material of the body.

Q1// A stiff rod 1m long, and 20mm x 20mm in cross section is subjected to a tensile force of 40kN. Determine the elongation of rod if the modulus of elasticity for the rod material is 200GPa (Giga Pascal)?

Given

$$L = 1\text{m long} = 1000\text{mm}$$

$$A = 20\text{mm} \times 20\text{mm} = 400\text{mm}^2$$

$$P = 40\text{kN} = 40 \times 10^3\text{N}$$

$$E = 200\text{GPa} = 200 \times 10^9\text{N/m}^2$$

$$= \frac{200 \times 10^9\text{N}}{1000 \times 1000\text{mm}^2} = 200 \times 10^3\text{N/mm}^2$$

$$\Delta L = \frac{PL}{AE}$$

$$= \frac{(40 \times 10^3\text{N})(1 \times 10^3\text{mm})}{(400\text{mm}^2)(200 \times 10^3\text{N/mm}^2)}$$

$$= 0.5\text{mm}$$

Q2// A hollow cylinder 2m long has an outside diameter of 50mm and inside diameter of 30mm. If the cylinder is carried a load of 25kN. Find the stress in the cylinder and also find the deformation of the cylinder. If the value of modulus of elasticity for the cylinder material is 100GPa?

Given

$$L = 2\text{m} = 2000\text{mm}$$

$$D = 50\text{mm}$$

$$d = 30\text{mm}$$

$$P = 25\text{kN} = 25 \times 10^3\text{N}$$

$$E = 100\text{GPa} = 100 \times 10^3\text{N/mm}^2$$

$$\sigma = \frac{P}{A}$$

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (50^2 - 30^2) = 1257.14\text{mm}^2$$

$$= \frac{25 \times 10^3\text{N}}{1257.14} = 19893\text{N/mm}^2$$

$$\Delta L = \frac{PL}{AE} = \frac{25 \times 10^3 \times 2000\text{mm}}{1257.14\text{mm}^2 \times 100 \times 10^3} = 0.398\text{mm} \approx 0.4\text{mm} \text{ Ans}$$

Q1) A load of 5 kN is raised to be raised with help of still wire find the minimum diameter of the still rod. If stress is not exist do 100 MPa?

Given

$$P = 5 \text{ kN} = 50 \times 10^3 \text{ N}$$

$$\sigma = 100 \times 10^6 \text{ N/m}^2$$

$$\frac{100 \times 10^6 \text{ N}}{10^3 \times 10^3 \text{ mm}^2} = 100 \text{ N/mm}^2$$

$$A = \frac{P}{\sigma} = \frac{5 \times 10^3}{100} = 50 \text{ mm}^2$$

$$\frac{\pi}{4} \times d^2 = d = \frac{4 \times 50}{\pi} = 7.979 \text{ mm}$$

$$\sigma = \frac{P}{A}$$

$$A = \frac{P}{\sigma}$$

→ Lateral strain! → It is defined as the ratio of change in depth to original depth (mathematically $e_l = \frac{\Delta d}{d}$)

→ Linear strain! → It is defined as the ratio of change in length to original length (mathematically $e_l = \frac{\Delta l}{l}$)

→ Poisson's ratio! → when deformation of the member as with in the elastic limit it is found that the ratio of the lateral strain to the linear strain, is a constance for a given material, the ratio is called poisson's ratio.

→ It is denoted by $\frac{1}{m}$ or μ .

$$\text{Mathematically } \frac{1}{m} = \frac{\text{lateral strain}}{\text{linear strain}}$$

→ Volumetric strain! → It is defined as the ratio of change in volume to the original volume of the body.

→ It is denoted by $e_v = \frac{\Delta V}{V}$

→ N.B

→ The rectangular section $e_v = e_b + e_c + e_d$

→ circular section $e_v = \frac{\delta l}{l} + \frac{2\delta d}{d}$

10// A brass rod 400cm & the diameter of 4cm is subjected to an axial load of 5000kg calculate change in length change in diameter and change in volume. Take young's modulus of the brass rod is $1 \times 10^6 \text{ kg/cm}^2$ and Poisson's ratio is 0.3?

Given - $l = 400 \text{ cm}$

$d = 4 \text{ cm}$

$P = 5000 \text{ kg}$

$E = 1 \times 10^6 \text{ kg/cm}^2$

$\mu = 0.3$

$\delta l, \delta d \& \delta V$

$e_l = \frac{\delta l}{l}, e_d = \frac{\delta d}{d}$

$A = \frac{\pi}{4} \times d^2$

$= \frac{\pi}{4} \times 4^2$

$= 12.56$

$e_l = \frac{e_d}{\mu} = 0.3 = \frac{\frac{\delta d}{d}}{\frac{\delta l}{l}} = \frac{0.3 \times 0.159}{400} = \frac{\delta d}{d}$

$\Rightarrow \delta d = \frac{0.3 \times 0.159 \times 4}{400} = 0.002$

$e_v = \frac{\delta l}{l} + \frac{2\delta d}{d} = \frac{0.159}{400} + 2 \left(\frac{0.002}{4} \right) = 0.001 \text{ Ans}$

$e_v = \frac{\delta V}{V} \Rightarrow V = A \times l$

$= V = 12.56 \times 400 = 5024 \text{ cm}^3$

$\delta V = 0.001 \times 5024 = 5.024 \text{ cm}^3 \text{ Ans}$

Q211 A still rod 1m long 20mm diameter is subjected to an axial load of 415 kN. find change in length change in diameter and change in volume of the rod. take modulus of elasticity for still rod is $2 \times 10^5 \text{ N/mm}^2$ & Poisson's ratio is $1/4$?

Given

$$L = 1\text{m} = 1000\text{mm}$$

$$d = 20\text{mm}$$

$$P = 415\text{ kN} = 415 \times 10^3 \text{ N}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\mu = 1/4$$

$$\delta L = ? \quad \delta d = ? \quad \delta V = ?$$

$$e_L = \frac{\delta L}{L}, \quad e_d = \frac{\delta d}{d}, \quad e_v = \frac{\delta V}{V}$$

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4} \times 20^2$$

$$= 157.08 \text{ mm}^2$$

$$e_L = \frac{e_d}{\mu}$$

$$\frac{1}{4} = \frac{e_d}{\frac{\delta L}{L}}$$

We know $\frac{\delta L}{L} = \frac{P \cdot L}{A \cdot E}$

$$\frac{\delta L}{L} = \frac{P \cdot L}{A \cdot E}$$

$$= \frac{415 \times 10^3 \times 1000}{157.08 \times 2 \times 10^5} = 57.325$$

$$= \frac{1}{4} \times \frac{57.325 \times 1000}{20} = \frac{\delta d}{d}$$

$$= \delta d = \frac{1}{4} \times 57.325 \times 20 = 0.072$$

$$e_l = \frac{\delta l}{l} + \frac{2\delta d}{d}$$

$$= \frac{5.7325}{4000} + 2\left(\frac{0.072}{20}\right)$$

$$= 0.022 \text{ ans}$$

$$e_v = \frac{\delta v}{v}$$

$$v = 4000 \times 15.70$$

$$= 62800 \text{ cm}^2$$

$$\delta v = 0.022 \times 62800$$

$$= 1381.600 \text{ cm}^3 \text{ ans}$$

Given

$$L = 4\text{m} = 4000\text{mm}$$

$$d = 20\text{mm}$$

$$P = 4\pi \text{KN} = 4\pi \times 10^3 \text{N}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$l = 1/4$$

$$\delta l = ?, \delta d = ?, \delta v = ?$$

$$e_l = \frac{\delta l}{l}, e_d = \frac{\delta d}{d}, e_v = \frac{\delta v}{v}$$

$$A = \pi/4 \times d^2$$

$$= \frac{\pi}{4} \times 20^2 = 314.159$$

We know

$$\delta l = \frac{P \cdot L}{A \cdot E}$$

$$= \frac{415 \times 10^3 \times 4000 \text{ mm}}$$

$$3141.159 \times 2 \times 10^5 \text{ N/mm}^2$$

$$= 2728.37 \text{ mm}$$

$$\frac{\delta l}{L} = \frac{2728.37}{4000}$$

$$= 0.6821$$

Elastic Constants

→ Young's modulus (E) :→ Young's modulus is defined as the ratio between simple stress to the strain.

Mathematically $E = \frac{\text{Stress } (\sigma)}{\text{Strain } (\epsilon)}$

→ Modulus of Rigidity: (C) or (G)

→ It is defined as the ratio of shear stress to the shear strain.

Mathematically $C = \frac{\text{Shear stress}}{\text{Shear strain}}$

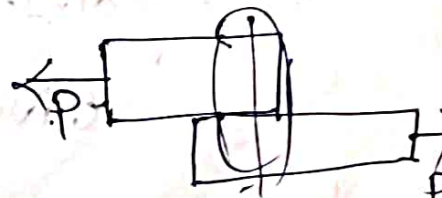
→ Shear Stress :→ when a section is subjected to two equal and opposite forces acting tangentially ~~across~~ ^{across} the resisting section as a result of which the body tends to shear off. ~~across~~ the section, the stress induced is called shear stress.

→ The corresponding strain is called shear strain.

→ Shear stress is denoted by τ or σ

→ and shear strain is denoted by ϕ

$$C = \frac{\tau}{\phi}$$



→ Bulk Modulus (K) is defined as the ratio between simple stress to the volumetric strain.

→ It is denoted by ' K '

→ Hence force mathematically $K = \frac{\text{simple stress } (\sigma)}{\text{volumetric strain } (\epsilon_v)}$

10.12.10.2022

→ Relation betⁿ young's modulus & modulus of Rigidity:

$$E = 2C \left(1 + \frac{1}{m} \right)$$

$$\text{or } C = \frac{M.E}{2 \cdot (m+1)}$$

Relation betⁿ young's modulus & Bulk modulus.

$$E = 3K \cdot \left(1 - \frac{2}{m} \right)$$

2// A material has a young's modulus of $1.25 \times 10^5 \text{ N/mm}^2$ and the poisons ratio is 0.25 calculate modulus of rigidity & bulk modulus.

$$\text{Given, } E = 1.25 \times 10^5 \text{ N/mm}^2$$

$$\mu = 0.25 = \frac{1}{4}$$

$$C = \frac{M.E}{2(m+1)} = \frac{4 \times 1.25 \times 10^5}{2(4+1)} \\ = 50 \times 10^3 \text{ N/mm}^2$$

$$\begin{aligned}
 E &= 3K \left(1 - \frac{2}{m} \right) \\
 \Rightarrow K &= \frac{E}{3} \left(\frac{m}{m-2} \right) \\
 &= \frac{1.25 \times 10^5}{3} \left(\frac{4}{4-2} \right) \\
 &= 83.
 \end{aligned}$$

Q2// A material has young modulus of $1.5 \times 10^5 \text{ N/mm}$ & poissons ratio 0.5 Calculate both bulk modulus & modulus of rigidity.

Given, $E = 1.5 \times 10^5$
 $\mu = 0.5 = \frac{1}{2}$

$$\begin{aligned}
 C &= \frac{M \cdot E}{2(M+1)} = \frac{2 \times 1.5 \times 10^5}{2(2+1)} \\
 &= 50 \times 10^3 \text{ N/mm}^2
 \end{aligned}$$

$$\begin{aligned}
 E &= 3K \left(1 - \frac{2}{m} \right) \\
 \Rightarrow K &= \frac{E}{3} \left(\frac{m}{m-2} \right) \\
 &= \frac{1.5 \times 10^5}{3} \left(\frac{2}{2-2} \right)
 \end{aligned}$$

where $e, K \& C \therefore E = \frac{9KC}{3K+C}$

Q3// A material has young modulus $1.5 \times 10^5 \text{ N/mm}^2$ and poisson ratio ~~is~~ is 0.23 calculate both C & K

Given

$$E = 1.5 \times 10^5 \text{ N/mm}^2$$

$$\mu = 0.23 = \frac{1}{4.347}$$

$$C = \frac{4.35 \times 1.5 \times 10^5}{2(4.35 \times 1)}$$

$$C = \frac{MXE}{2(m+1)}$$

$$= 75000 \text{ N/mm}^2$$

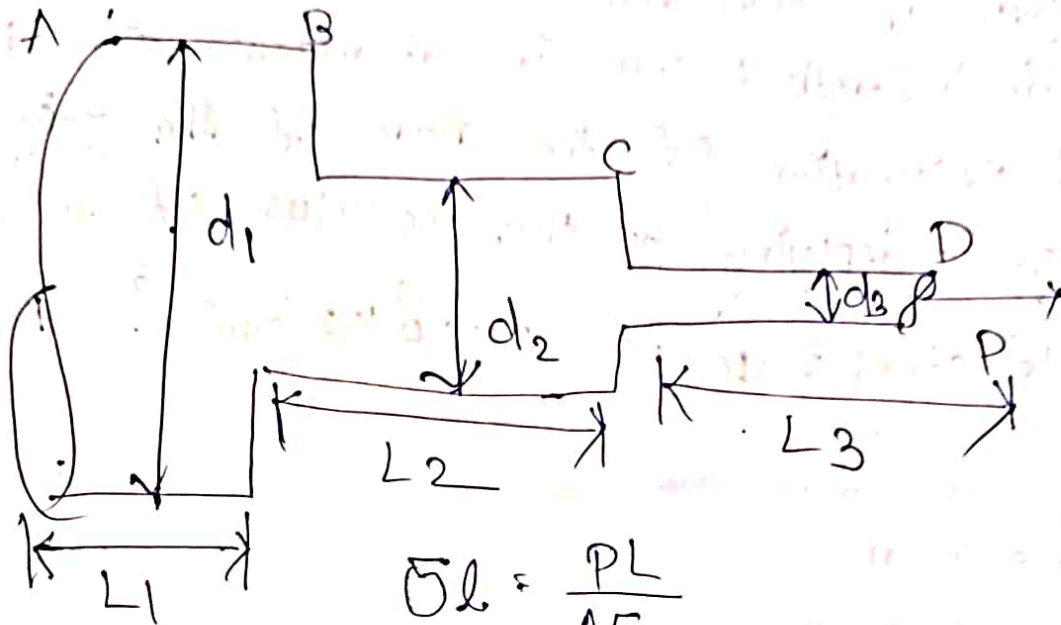
$$E = 3K \left(1 - \frac{2}{m}\right)$$

$$K = \frac{E}{3} \left(\frac{m}{m-2}\right)$$

$$= \frac{1.5 \times 10^5}{3} \left(\frac{4.35}{4.35-2}\right)$$

$$= 301.38 \text{ N/mm}^2 \quad \underline{\underline{\text{Ans}}}$$

Bars in Analysis of Varying Section:-



$$\delta L = \frac{PL}{AE}$$

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3$$

$$\delta L_1 = \frac{PL_1}{A_1 E}$$

$$\delta L_2 = \frac{PL_2}{A_2 E}$$

$$\delta L_3 = \frac{PL_3}{A_3 E}$$

So

$$\delta L = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E}$$

elongation of bar

$$\delta L = \frac{P}{E} \left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right)$$

Q1/ A mild steel bar 1500 mm long 30 mm of diameter, for 800 mm length of steel bar for 25 mm diameter and 4500 mm in length 20 mm in diameter. Calculate the total elongation of the bar if the axial pull of 5000 kg is applying & the modulus of elasticity of the mild steel bar is $2 \times 10^6 \text{ kg/cm}^2$?

Given

$$L_1 = 1500 \text{ mm}$$

$$d_1 = 30 \text{ mm}$$

$$L_2 = 800 \text{ mm}$$

$$d_2 = 25 \text{ mm}$$

$$L_3 = 4500 \text{ mm}$$

$$d_3 = 20 \text{ mm}$$

$$P = 5000 \text{ kg}$$

$$E = 2 \times 10^6 \text{ kg/cm}^2$$

$$\Delta L = \frac{P}{E} \left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right)$$

$$A_1 = \frac{\pi}{4} \times d_1^2$$

$$= \frac{\pi}{4} \times 30^2 = 706.9 \text{ mm}^2$$

$$A_2 = \frac{\pi}{4} \times 25^2 = 490.9 \text{ mm}^2$$

$$A_3 = \frac{\pi}{4} \times 20^2 = 314.1 \text{ mm}^2$$

$$\Delta L = \frac{P}{E} \left(\frac{1500}{706.9} + \frac{800}{490.9} + \frac{4500}{314.1} \right)$$

$$= \frac{5000}{2 \times 10^6} \left(\frac{1500}{706.9} + \frac{800}{490.9} + \frac{4500}{314.1} \right) = 0.452 \text{ mm}$$

Q. About 30mm diameter is subjected to a pull of 60kN the major extension on gauge length of 200mm is change in length of 0.09mm and the change in diameter 0.0039mm calculate the Poisson's ratio and the value of Elastic Constant?

Given

$$d = 30 \text{ mm}$$

$$P = 60 \text{ kN} = 60 \times 10^3 \text{ N}$$

$$L = 200 \text{ mm}$$

$$\delta L = 0.09 \text{ mm}$$

$$\delta d = 0.0039 \text{ mm}$$

$$\mu = \frac{\frac{\delta d}{d}}{\frac{\delta L}{L}} = \frac{\delta L}{L} \cdot \frac{0.09}{200} = 4.5 \times 10^{-4} \text{ mm}$$

$$= \frac{\delta d}{d} = \frac{0.0039}{30} = 1.3 \times 10^{-4} \text{ mm}$$

$$\mu = \frac{1.3 \times 10^{-4}}{4.5 \times 10^{-4}} = 0.288$$

$$\frac{1}{m} = \frac{1}{0.28} = 3.57$$

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 30^2 = 706.9 \text{ mm}^2$$

$$E = \frac{P}{A} = \frac{60 \times 10^3}{706.9} = 84.86 \times 10^3 \text{ N/mm}^2$$

$$C = \frac{m \cdot E}{2(m+1)} = \frac{3.57 \times 188.62 \times 10^3}{2(3.57+1)} = 73673.238 \text{ N/mm}^2$$

$$K = \frac{m \cdot E}{3(m-2)} = \frac{3.57 \times 188.62 \times 10^3}{3(3.57-2)} = 142966.7516 \text{ N/mm}^2$$

Q2/ A bar $30 \times 30 \text{ mm}$ & 250 mm long is subjected to a pull of 90 kN in the direction of its length. The extension of bar core found to be 0.125 mm . The diameter in lateral diameter is found to be 0.00375 mm . Find young's modulus of rigidity and the Poisson's ratio for the material bar.

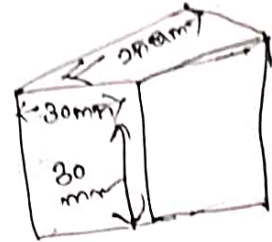
Given $A = 30 \times 30 \text{ mm}^2$

$L = 250 \text{ mm}$

$P = 90 \text{ kN} = 90 \times 10^3 \text{ N}$

$\Delta L = 0.125 \text{ mm}$

$\Delta d = 0.00375 \text{ mm}$



Find $= E, K, C, \nu$

$E = \frac{\sigma}{e}$

where $\sigma = \frac{P}{A} = \frac{90 \times 10^3}{30 \times 30} = 100$

$e = \frac{\Delta L}{L} = \frac{0.125}{250} = 5 \times 10^{-4}$

$(E) = \frac{100}{5 \times 10^{-4}} = 200000 = 200 \times 10^3 \text{ N/mm}^2$

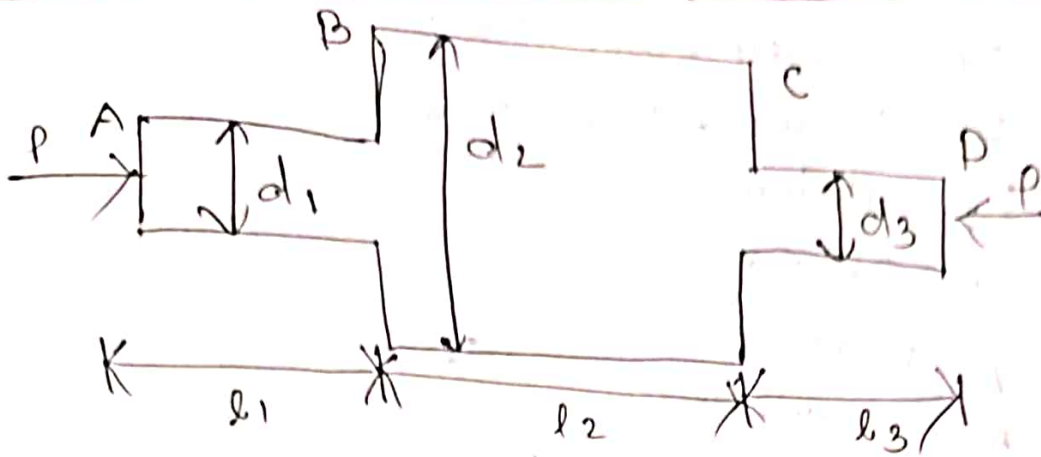
$e_d = \frac{\Delta d}{d} = \frac{0.00375}{30} = 1.25 \times 10^{-4}$

$(\nu) = \frac{\frac{\Delta d}{d}}{\frac{\Delta L}{L}} = \frac{1.25 \times 10^{-4}}{5 \times 10^{-4}} = 0.25$

$C = \frac{ME}{2(m+1)} = \frac{4 \times 200000}{2 \times 4 + 1} = 80 \times 10^3 \text{ N/mm}^2$

$E = \frac{E}{3} \left(\frac{m}{m-2} \right) = \frac{200000}{3} \left(\frac{4}{4-2} \right) = 13 \times 10^4$

Stresses in the Bar In Different section:-



The modulus of elasticity (E) is different for the different section in such case the total deformation (δl).

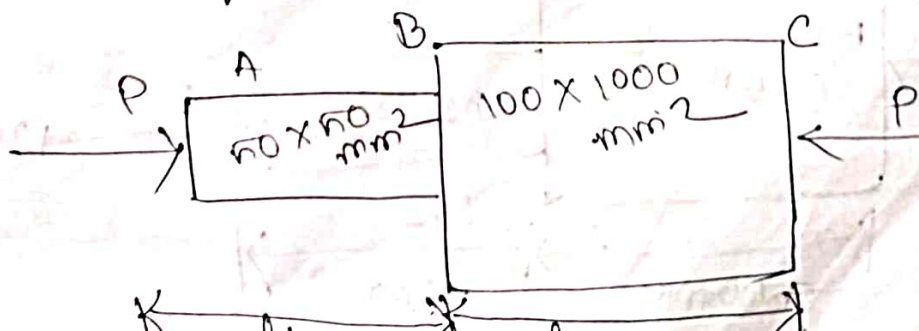
$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$\delta l_1 = \frac{Pl_1}{A_1 E_1} \quad \text{if} \quad \delta l_2 = \frac{Pl_2}{A_2 E_2} \quad + \quad \delta l_3 = \frac{Pl_3}{A_3 E_3}$$

$$\delta l = \frac{Pl_1}{A_1 E_1} + \frac{Pl_2}{A_2 E_2} + \frac{Pl_3}{A_3 E_3}$$

$$\boxed{\delta l = P \left(\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} + \frac{L_3}{A_3 E_3} \right)}$$

The member fixed by connecting a steel bar to an aluminium bar as shown in figure assuming that the bars are prevented from buckling sideways calculate the magnitude of force (P) that will cause the total length of the member to decrease by 0.25 mm the values of elastic modulus for steel & aluminium are 210 GPa & 70 GPa respectively.



Given:-

$$\delta l = 0.25 \text{ mm}$$

$$E_1 = 210 \text{ GPa} = 210 \times 10^3 \text{ N/mm}^2$$

$$E_2 = 70 \text{ GPa} = 70 \times 10^3 \text{ N/mm}^2$$

$$L_1 = 300 \text{ mm}$$

$$L_2 = 380 \text{ mm}$$

$$A_1 = 50 \times 50 \text{ mm}^2$$

$$A_2 = 100 \times 100 \text{ mm}^2$$

$$\delta l = P \left(\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} \right)$$

$$0.25 = P \left(\frac{300}{(50 \times 50 \times 210 \times 10^3)} + \frac{380}{(100 \times 100 \times 70 \times 10^3)} \right)$$

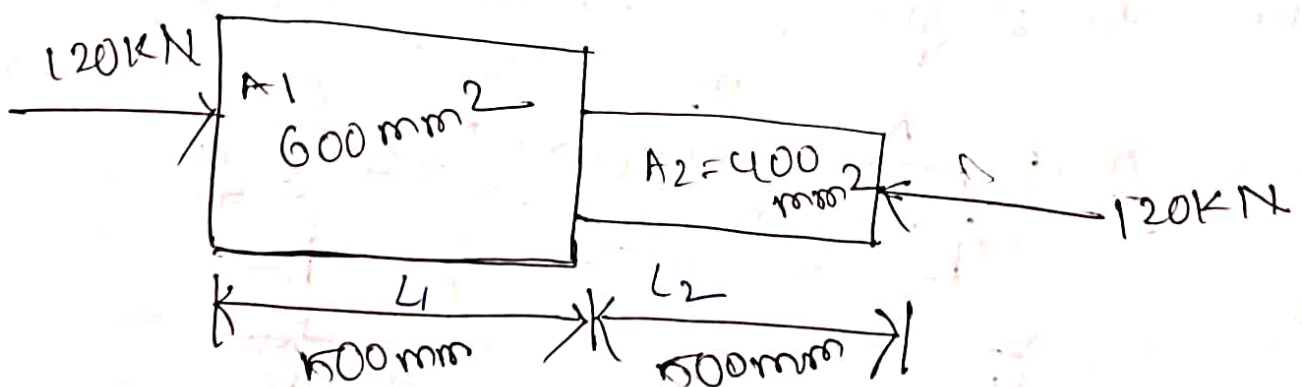
$$= \frac{0.25}{\frac{300}{50 \times 50 \times 210 \times 10^3} + \frac{380}{100 \times 100 \times 70 \times 10^3}}$$

$$= \frac{0.25}{\frac{300}{50 \times 50 \times 210 \times 10^3} + \frac{380}{100 \times 100 \times 70 \times 10^3}}$$

$$= 224.35 \times 10^3 \text{ N}$$

$$P = 224.35 \text{ kN} \quad \underline{\underline{\text{Ans}}}$$

Q2) A steel bar as shown in figure subjected to a tensile force of 120 kN calculate the elongation of the bar. Take the elasticity modulus of first material is 200 GPa and another material is 100 GPa.



Given

$$P = 120 \text{ kN} = 120 \times 10^3 \text{ N}$$

$$L_1 = 500 \text{ mm}$$

$$L_2 = 500 \text{ mm}$$

$$A_1 = 600 \text{ mm}^2$$

$$A_2 = 400 \text{ mm}^2$$

$$E_1 = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$$

$$E_2 = 100 \text{ GPa} = 100 \times 10^3 \text{ N/mm}^2$$

Sl.?

$$\delta L = P \left(\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} \right)$$

$$= 120 \times 10^3 \left(\frac{500}{600 \times 200 \times 10^3} + \frac{500}{400 \times 100 \times 10^3} \right) \text{ mm}$$

$$= 2 \text{ mm}$$

Stresses in the Bars of composite structure :-

composite structure

A Bar made up of two or more different materials joined together is known as composite Bar or Composite structure.

Consider a composite Bar made up of two different materials as shown in figure. Let P is ^{total} force or load on the Bar.

L_1 = Length of the Bar 1.

L_2 = Length of the Bar 2.

A_1 = Area of the Bar 1.

E_1 = Modulus of Elasticity of Bar 1.

P_1 = Load share by Bar 1.

Similarly A_2 , E_2 & P_2 considering the values of Bar 2.

We know the total load $P = P_1 + P_2$

We know $\sigma = \frac{P}{A}$

Stress in the Bar 1 $\sigma_1 = \frac{P_1}{A_1}$

Strain in Bar 1 $e_1 = \frac{\sigma_1}{E_1}$

$$\therefore E = \frac{\sigma}{e}$$

$$e_1 = \frac{\frac{P_1}{A_1}}{E_1} = \frac{P_1}{A_1 E_1}$$

→ Therefore elongation $\Delta l_1 = e_1 l_1$ $\therefore e_1 = \frac{\Delta l_1}{l_1}$

$$\Delta l_1 = \frac{P_1 l_1}{A_1 E_1} \quad \text{--- eqn ①}$$

Similarly Elongation Bar 2 $\Delta l_2 = \frac{P_2 l_2}{A_2 E_2}$ → eqn ②

Since both the elongation are equal therefore equating eqn ① & eqn ② $\therefore$

$$\Delta l_1 = \Delta l_2 = \frac{P_1 l_1}{A_1 E_1} = \frac{P_2 l_2}{A_2 E_2}$$

$$\therefore l_1 = l_2 = l$$

$$\frac{P_1}{A_1 E_1} = \frac{P_2}{A_2 E_2}$$

$$P_1 = \frac{P_2}{A_2 E_2} \times A_1 E_1 \quad \text{--- eqn - (3)}$$

$$\Rightarrow \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

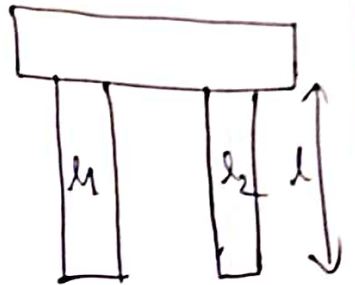
From these eqn we can find the load share by the different material, we have also seen the above load is same them.

$$\Rightarrow \sigma_1 = \frac{E_1}{E_2} \times \sigma_2$$

$$\Rightarrow \sigma_2 = \frac{E_2}{E_1} \times \sigma_1$$

→ From the above eqn we can find the stress in different material. then we also know that total load $(P) = P_1 + P_2 = \sigma_1 A_1 + \sigma_2 A_2$

→ The ratio of elasticity of two material is known as modulus ratio. It is denoted by the letter (m) .



Q2// A reinforced concrete circular section of 50,000 mm² cross-sectional area carries six reinforcing Bar whose total area is 500 mm². Find the share load (P), the column can carry, if the concrete is not to be stressed more than 3.5 MPa. Take modular ratio for the steel & concrete as 18?

Given

$$A = 50000 \text{ mm}^2 = 500 \text{ mm}^2$$

$$A_s = 500 \text{ mm}^2$$

$$A_c = 49,500 \text{ mm}^2$$

$$f_c = 3.5 \text{ MPa} = 3.5 \text{ N/mm}^2$$

$$f_s = \frac{E_s}{E_c} \times f_c$$

$$\frac{E_s}{E_c} = 18$$

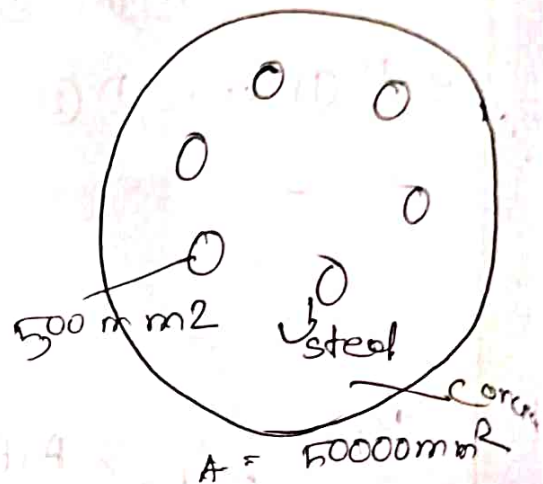
$$f_s = \frac{E_s}{E_c} \times f_c$$

$$= 18 \times 3.5 = 63 \text{ N/mm}^2$$

$$P = (f_s \cdot A_s) + (f_c \cdot A_c)$$

$$= (63 \times 500) + (3.5 \times 49,500)$$

$$= 204750 \text{ N}$$



P = ?

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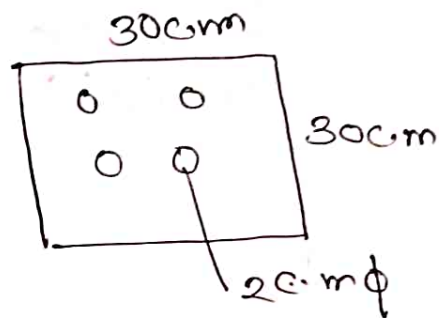
A RCC column of $30\text{ cm} \times 30\text{ cm}$ has four reinforced bar of 2 cm dia, calculate the ~~same~~ load carrying by the column If the avle stress of concrete is 40 kg/cm^2 Take youngs modulus for steel is 15 times of youngs modulus of concrete?

Given

$$f_c = 40\text{ kg/cm}^2$$

$$E_s = 15 E_c$$

$$\Rightarrow \frac{E_s}{E_c} = m = 15$$



$$P = f_s A_s + f_c A_c$$

$$A = 30\text{ cm} \times 30\text{ cm} = 900\text{ cm}^2$$

$$A_s = 4 \times \frac{\pi}{4} \times 2^2 = 12.57\text{ cm}^2$$

$$A_c = 900 - 12.57 = 887.43\text{ cm}^2$$

$$f_s = \frac{E_s}{E_c} \times f_c$$

$$= 15 \times 40 = 600\text{ kg/cm}^2$$

$$P = f_s A_s + f_c A_c$$

$$= 600 \times 12.57 + 40 \times 887.43$$

$$= 43039.2 = 43.03 \times 10^3\text{ kg} \quad \underline{\underline{\text{Ans}}}$$

2) A copper rod and 1 steel rod together support a rigid uniform beam weighting of P as shown in figure the stress in copper and steel are not exist 60 N/mm^2 & 120 N/mm^2 respectively. find the magnitude of load that can be safely supported the young's modulus for steel, is twice that of copper?

Given $A_C = (40 \times 40) \times 2 = 3200 \text{ mm}^2$

$A_S = 50 \times 50 = 2500 \text{ mm}^2$

$l_C = 150 \text{ mm}$

$l_S = 250 \text{ mm}$

$f_C = 60 \text{ N/mm}^2$

$f_S = 120 \text{ N/mm}^2$

$E_S = 2E_C$

$\Rightarrow \frac{E_S}{E_C} = 2$

$\Rightarrow$ we know,

$e = \frac{\delta l}{l}$

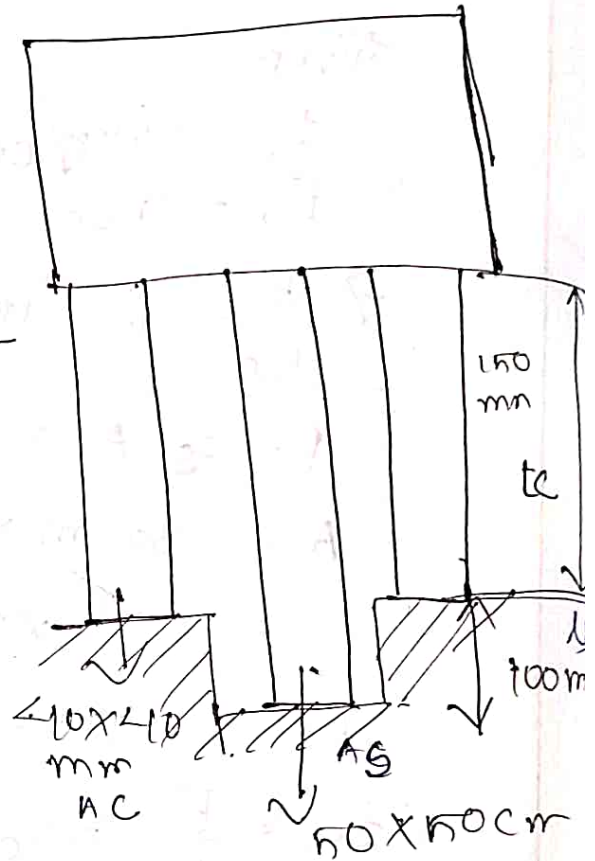
$\Rightarrow \delta l_S = e_S \times l_S$

$\Rightarrow \delta l_C = e_C \times l_C$

$\Rightarrow \delta l_S = \delta l_C$

$\Rightarrow e_S \times l_S = e_C \times l_C$

$\Rightarrow \frac{e_S}{e_C} = \frac{l_C}{l_S} = \frac{150}{250} = 0.6 = \frac{e_S}{e_C}$



$$E_s = \frac{\sigma_s}{\epsilon_s}, E_c = \frac{\sigma_c}{\epsilon_c}$$

$$\sigma_s = E_s \cdot \epsilon_s, \sigma_c = E_c \cdot \epsilon_c \rightarrow \text{eqn (2)}$$

$$\epsilon_s = \epsilon_c \rightarrow \text{eqn (1)}$$

$\Rightarrow$ Dividing eqn (1) & (2)

$$\frac{\sigma_s}{\sigma_c} = \frac{E_s \cdot \epsilon_s}{E_c \cdot \epsilon_c} = 2 \times 0.6 = 1.2$$

$$\Rightarrow \sigma_s = 1.2 \times \sigma_c$$

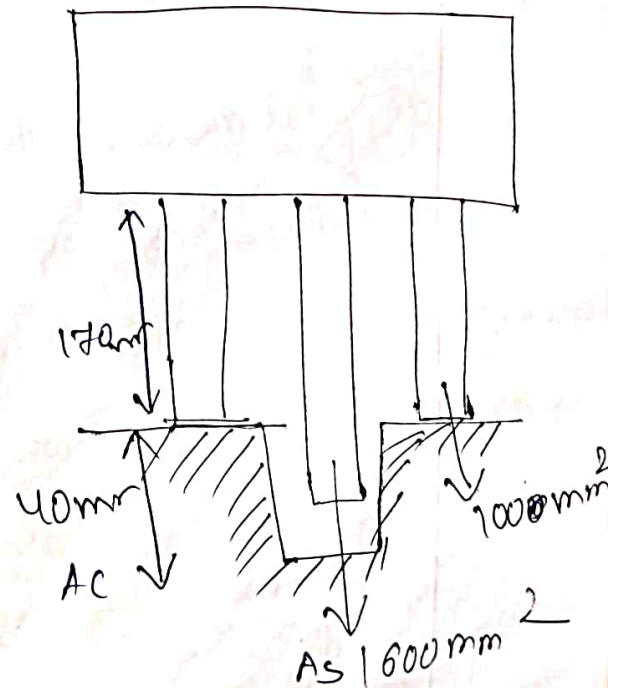
$$\Rightarrow \sigma_s = 1.2 \times 60 = 72 \text{ N/mm}^2$$

$$P = \sigma_c A_c + \sigma_s A_s$$

$$= 60 \times 3200 + 72 \times 2500$$

$$= 37.2 \times 10^4 \text{ N}$$

3// Two copper rod and steel rod together support a load of 250 kN as shown figure find the stress in the rod take $E_s = 2 \times 10^5 \text{ N/mm}^2$ and $E_c = 1 \times 10^5 \text{ N/mm}^2$.



Given

$$P = 250 \text{ kN}$$

$$E_s = 2 \times 10^5 \text{ N/mm}^2$$

$$E_c = 1 \times 10^5 \text{ N/mm}^2$$

$$A_c = 40 \text{ mm}$$

$$A_s = 1600 \text{ mm}^2$$

→ Principle of

Temperature Stress:-

1. Temp. stress $-(\sigma) = \alpha T E$

where σ = Temp. stress

α = Temp expansion in $1/^\circ\text{C}$

T = Temperature

E = Young's modulus

2. Temp. force $-(P) = \alpha T E A$

where P = Temp. force or load

A = Cross sectional area

3. yield temp. strain $(e) = \alpha T - \frac{\Delta}{L}$

4. yield temp. stress $= \left(\alpha T - \frac{\Delta}{L} \right) E$

5. yield temp. force $= \left(\alpha T - \frac{\Delta}{L} \right) E A$

3// A copper bar 20mm dia and 100cm long is firmly held between two end grips. it is then heated to 100°C . calculate the push exerted by the clamp through in the heated state. assume young's modulus for copper $1.1 \times 10^6 \text{ kg/cm}^2$. and α is equal to temperature expansion coefficient for copper is $1.8 \times 10^{-5} / ^\circ\text{C}$?

Given

$d = 20 \text{ mm} \times 20 \text{ mm}$ $E = 1.1 \times 10^6 \text{ kg/cm}^2$

$L = 200 \text{ cm}$ $\alpha = 1.8 \times 10^{-5} / ^\circ\text{C}$

$T_i = 100^\circ\text{C}$, $T_f = 0^\circ$

we know

$$\delta = \alpha T E$$

$$T = T_f - T_i = 100^\circ - 0^\circ = 100^\circ \text{C}$$

$$= 1.8 \times 10^{-5} \times 100 \times 1.10 \times 10^6$$

$$= 1980 \text{ kg/cm}^2$$

$$P = \alpha T E A$$

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4} \times 20^2 = 3.141 \text{ cm}^2$$

$$= 314.15 \text{ mm}^2$$

$$= 1.8 \times 10^{-5} \times 100 \times 1.10 \times 10^6 \times 3.141$$

$$= 6219.181 \text{ kg}$$

Q2 // A rod of steel is 20m long, a temp. of 20°C now it is heated up to 65°C find the temp. stress produce when the rod is allow to expand 2mm only take α is 12×10^{-6} & $E = 2 \times 10^6 \text{ kg/cm}^2$

Given

$$L = 20 \text{ m long} = 2000 \text{ cm}$$

$$T = T_f - T_i = 65 - 20 = 45^\circ \text{C}$$

$$\alpha = 12 \times 10^{-6} / ^\circ \text{C}$$

$$E = 2 \times 10^6 \text{ kg/cm}^2$$

$$\Delta = 2 \text{ mm} = 0.2 \text{ cm}$$

temp. stress : ~~($\frac{\Delta}{L}$)~~ $\alpha T E$

$$\left(\alpha T - \frac{\Delta}{L} \right) E$$

$$= \left(12 \times 10^{-6} \times 150 - \frac{0.2}{2000} \right) \times 2 \times 10^6$$

$$= 880 \text{ kg/cm}^2$$

2/ A steel rod 20mm dia and 6m long is connected through two grips one at each end at a temperature of 120°C . Find the pull when the temp. falls to 40°C . (i) If the ends do not yield

(ii) If the ends are yield by 0.11cm.

Take young's modulus for the steel rod $2 \times 10^6 \text{ kg/cm}^2$ & α is $12 \times 10^{-6} / ^\circ\text{C}$?

Given $L = 6\text{m} = 600\text{cm}$
 $d = 20\text{mm} = 2\text{cm}$

$$T_f = 120^\circ\text{C}$$

$$T_i = 40^\circ\text{C}$$

$$T = T_f - T_i = 120 - 40 = 80^\circ\text{C}$$

$$\Delta = 0.11\text{cm}$$

$$E = 2 \times 10^6 \text{ kg/cm}^2$$

$$\alpha = 12 \times 10^{-6} / ^\circ\text{C}$$

$$A = \frac{\pi}{4} \times 2^2 = 3.142 \text{ cm}^2$$

$$\boxed{\text{Tem force} = \alpha T E A}$$

$$= 12 \times 10^6 \times 80 \times 2 \times 10^6 \times 3.142$$

$$= 6032.64 \text{ kg}$$

$$\boxed{\gamma_{\text{ield temp. force}} = \left(\alpha T - \frac{\Delta}{L} \right) E A}$$

$$= \left(12 \times 10^6 \times 80 - \frac{0.11}{600} \right) 2 \times 10^6 \times 3.142$$

$$= 41880.57 \text{ kg}$$

Ans

→ Meha Mechanical Properties of material →

→ The characteristics of material that describe the behavior under the action of external loads are referred as its mechanical properties. The common mechanical properties are follows.

Strength

- It is defined as the ability of a material to ~~resist~~ ^{resist} loads without failure.
- It is usually expressed or measured in terms of maximum load per unit area (i.e. maximum stress or ultimate strength) that a material can withstand failure and it varies according to the type of loading. Further the strength is divided into three types they are

- * Tensile strength! - The tensile strength or tenacity is defined as defined as the ability of material to resist a stretching (tensile) load without fracture.
- * Compressive strength! - The ability of a material to resist squeezing (compressive) load without fracture is called compressive strength.
- * Shear strength! - The ability of a material to resist transverse loads i.e. loads tending to separate (or cut) the material is called shear strength.

→ Rigidity! → Rigidity is defined as the property exhibited by the solid to change in its shape. That is when an external force is applied to the solid material, there won't be any change in the shape. This shows that the particles are closely packed and the attraction between these particles is very strong.

→ Elasticity! → The ability of materials to deform under load and return to its original shape when the load is removed is called elasticity.

→ Plasticity! → The ability of material to deform under load and return to its original shape when the load is removed is called elasticity.

→ Plasticity! → The ability of a material to deform under load and retain its new shape when the load is removed is called plasticity.

→ Compressibility! → Compressibility may be defined as the ratio of change in volume of the fluid to the original volume. It can be mathematically expressed as

→
$$\left(\frac{\text{Change in volume}}{\text{original}} \right) \times 100.$$

→ It is often represented in percentage that is why 100.

→ Hardness! →

→ It is defined as the ability of a material to resist scratching or indentation by another hard body. Hardness is directly related to strength.

→ Toughness! → It is defined as the ability of the material to absorb energy ~~scratching or indentation~~ by another hard body. ~~Hardness~~ is directly related to strength.

→ Toughness! → It is defined as the ability of the material to absorb energy up to fracture during the plastic deformation. Toughness of a metal offers the resistance to breaking when force is applied.

→ Stiffness! → It is the ability of material to resist deformation or deflection under load.

→ Within the elastic limit, stiffness is measured by the modulus of elasticity.

→ Brittleness! - ~~It is the ability of material to~~
It is property of sudden fracture without any visible permanent deformation.

→ Ductility! → It is the ability of a material to be deformed plastically without rupture under tensile load. Due to this property material can drawn out in to fine wire without fracture.

→ Malleability! → It is the ability of a material to be deformed plastically without rupture under compressive load.

→ Due to this property metals are hammered and rolled in to thin sheets.

→ Creep: → The slow and progressive deformation of material with time at constant stress is called creep.

→ Fatigue: →

→ Failure of material under repeated or reversal stress is called fatigue. Machine parts are frequently subjected to varying stress and it is important to know the strength of materials in such conditions.

→ The maximum stress at which the material will operate indefinitely without failure ~~strength of materials in such conditions~~

→ It is the endurance limit or fatigue limit.

→ Tenacity: → Tenacity is defined as the specific stress corresponding with the maximum force on a force / extension curve.

→ Durability: → Durability may be defined as the ability of a structure to remain fit for intended or foreseen use throughout its design working life, with an appropriate level of maintenance.

→ Resilience: → It is property of material to absorb energy and to resist shock and impact loads, It is measured by the amount of energy absorbed per unit volume within the elastic limit.

→ stress strain diagram!→

→ Suppose that a metal specimen be placed in tension or compression testing machine.

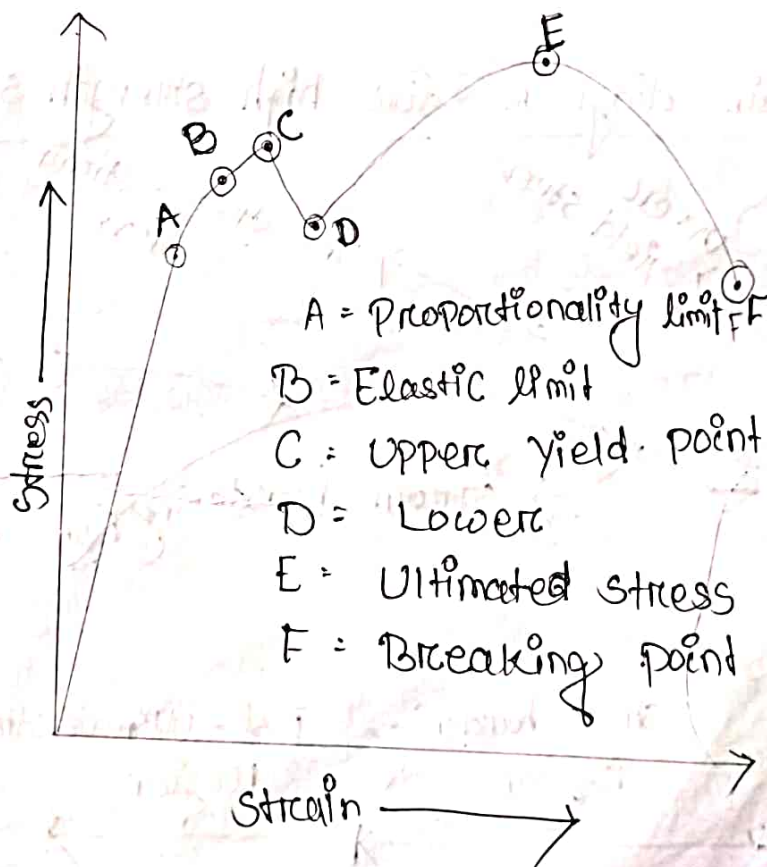
→ As the axial load is gradually increased in increments the total elongation over the gauge length is measured at each increment of the load and this is continued until failure of the specimen takes place.

→ Knowing the original cross-sectional area and length of the specimen, the normal stress σ and the strain E can be obtained.

→ The graph of these quantities with the stress σ along the y-axis and the strain E along the x-axis is called the stress-strain diagram.

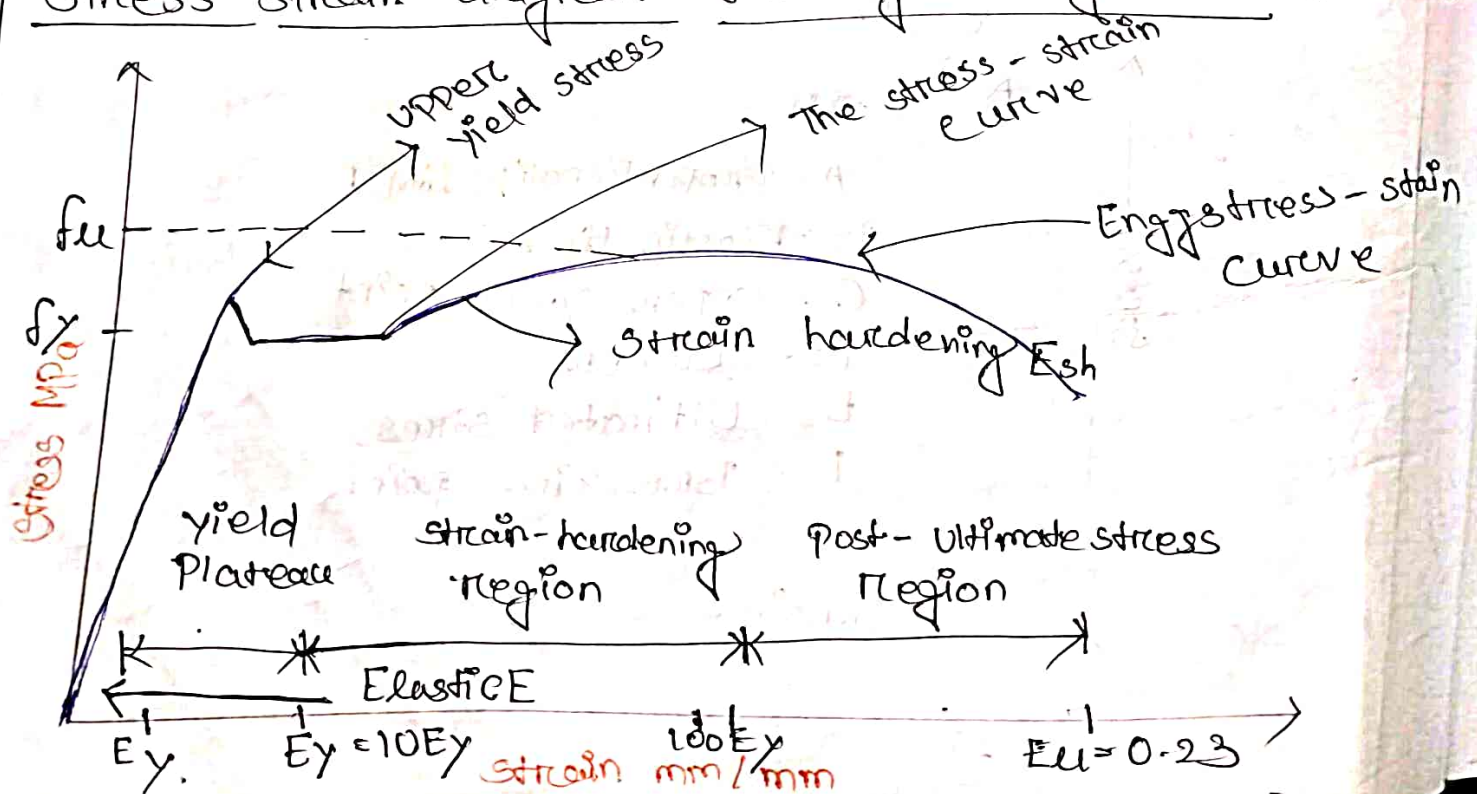
→ The stress strain diagram differs for various materials

* Stress strain diagram for mild steel :-



- Limit of proportionality! → (A)! → Proportionality limit is the maximum stress up to which the strain is directly proportional to stress.
- Elastic limit (B)! → Elastic limit is the maximum stress at which the material regain its deformation after removing the load.
- Upper yield point (C)! → Upper yield point is the point at which material yields and yield stress is the lowest stress at which material is appreciably deformed without increasing the load.
- Lower yield point (D)! → At this stage the stress remains same but strain increase for some time.
- Ultimate stress (E)! → Ultimate point is the maximum stress that a material can withstand without under tensile load.
- Breaking point (F)! → Breaking point is corresponding stress at which the failure of specimen takes place.

* Stress strain diagram for high strength steel!:-

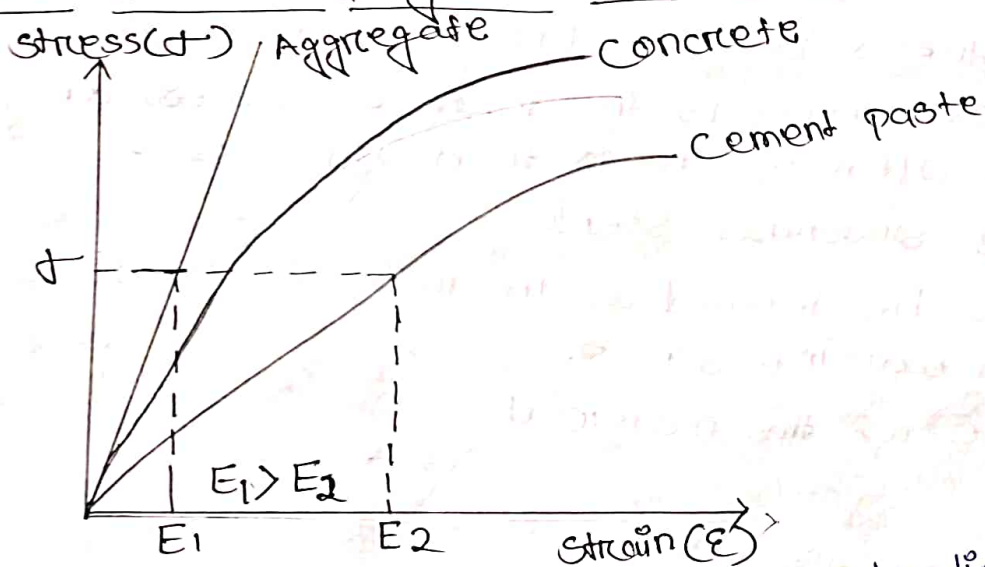


→ High strength steels, due to their specific microstructure, do not show a sharp yield point but rather they yield continuously as shown in figure.

→ For such steels the yield stress is always taken as the stress at which a line at 0.2% strain, parallel to the elastic portion, intercepts the stress strain curve. This is shown in figure.

→ The nominal stress or the engineering stress is given by the load divided by the original area. Similarly the engineering strain is taken as the ratio of the change in length to original length

★ stress strain diagram for concrete



→ The stress strain curve of concrete lies between those of the aggregate and the cement paste. However this relationship is non-linear over the most of the range.

→ Factor of safety

→ It is defined as the ratio of the ultimate stress to the working stress.

$$\text{Factor of safety, FOS} = \frac{\sigma_u}{\sigma_s}$$

→ The value of factor of safety is decided by designers by experience and based on reliability required.

→ In practice engineering components are designed with a margin of safety.

→ The working stress must be less than the stress which causes fracture or failure of the component.

→ working stress is also known as (design stress).

→ Factor of safety is the ratio of ~~stress at yield~~ between ultimate stress to design stress.

→ For the structural steel

→ It can be defined as the ratio of stress at yield point to working stress.

→ The type of the material

→ Load

→ Working conditions.

→ If stress continues, it surpasses its ultimate strength.

→ The ultimate strength is the greatest stress the object is subjected to when it is about to break.

→ The breaking point for a brittle material is just beyond its ultimate strength.

→ For a ductile material, after passing the ultimate strength, the material thins and stretches at a lower stress level. *Ductile*

Principle of superposition: Sometimes a body is subjected to a number of forces acting on it at ^{different} ~~adist~~ as well as at some other section along the length of the body.

In such case the forces are split up and their effects are considered on individual section.

The resulting deformation of the body is equal to the algebraic sum of the deformation of the individual section.

Such a principle of finding out the result of deformation is called the principle of superposition.

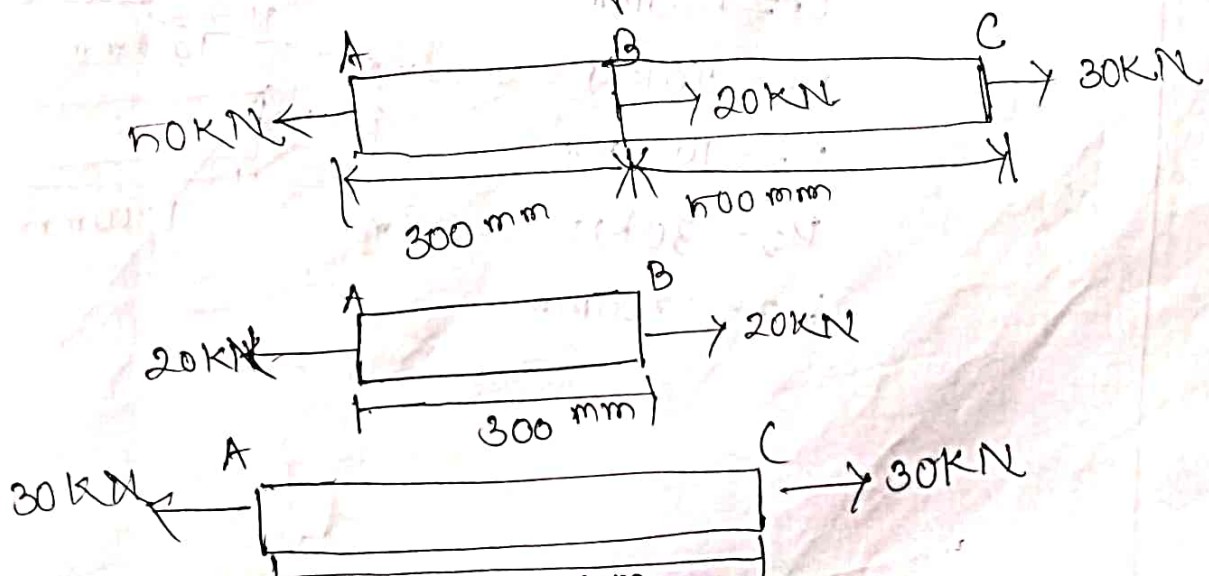
$$\Delta l = \frac{1}{AE} (P_1 l_1 + P_2 l_2 + P_3 l_3)$$

where

P_1, P_2 & P_3 are forces acting section 1, 2 & 3

l_1, l_2 & l_3 are length of the section 1, 2 & 3

Q. A steel bar of crosssectional area 200 mm^2 is loaded as shown in figure find the change in length of the bar take ~~modulus~~ E as 200 GPa ?



$$\delta l = \frac{1}{AE} (P_1 l_1 + P_2 l_2)$$

Given:

$$P_1 = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

$$L_1 = 300 \text{ mm}$$

$$P_2 = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$L_2 = 800 \text{ mm}$$

$$A = 200 \text{ mm}^2$$

$$E = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$$

$$= \frac{1}{200 \times 200 \times 10^3} (20 \times 10^3 \times 300 + 30 \times 10^3 \times 800)$$

$$= 0.75 \text{ mm} \quad \underline{\text{Ans}}$$

Q2/ A brass bar having crosssectional area of 500 mm^2 is subjected to axial force as shown. In figure find the total elongation the bar take AS

$$E = 80 \text{ GPa} ?$$

Given

$$A = 500 \text{ mm}^2$$

$$E = 80 \text{ GPa} = 80 \times 10^3 \text{ N/mm}^2$$

$$P_1 = 80 \text{ kN} = 80 \times 10^3 \text{ mm}$$

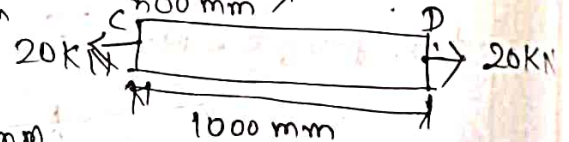
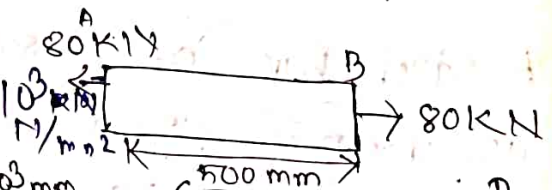
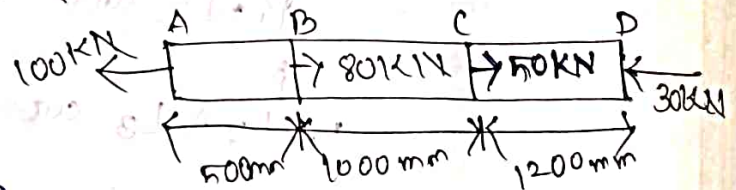
$$L_1 = 500 \text{ mm}$$

$$P_2 = 20 \text{ kN} = 20 \times 10^3 \text{ mm}$$

$$L_2 = 1000 \text{ mm}$$

$$P_3 = 30 \text{ kN} = 30 \times 10^3 \text{ mm}$$

$$L_3 = 1200 \text{ mm}$$



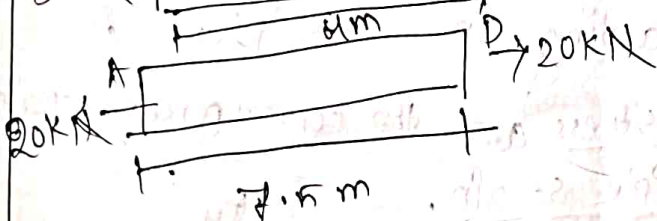
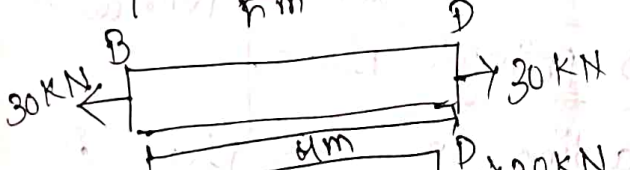
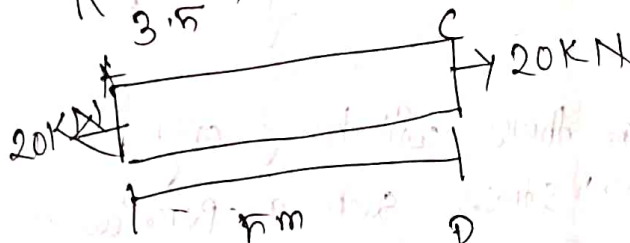
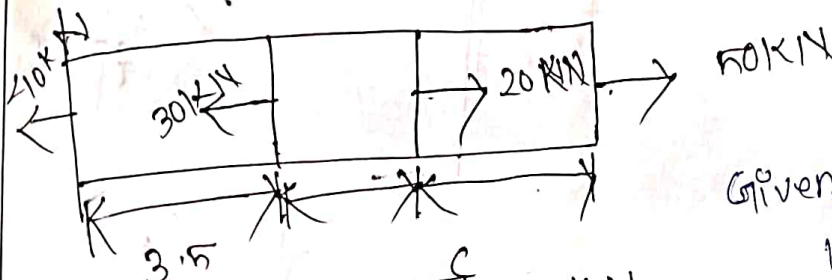
$$\Delta L = \frac{1}{AE} (P_1 L_1 + P_2 L_2 + P_3 L_3)$$

$$= \frac{1}{500 \times 80 \times 10^3} (80 \times 10^3 \times 20 \times 10^3 -$$

$$= \frac{1}{500 \times 80 \times 10^3} (80 \times 10^3 \times 500 + 20 \times 10^3 \times 1000 + 30 \times 10^3 \times 1200)$$

$$= 0.6 \text{ mm}$$

Q2/ A copper rod ABCD of 800 mm^2 crosssectional area and 7.5 m long is subjected to forces as shown in figure, find the elongation of bar take E for the copper is 100 GPa .



Given = $E = 100 \text{ GPa}$
 $100 \times 10^3 \text{ N/mm}^2$

$A = 800 \text{ mm}^2$

$P_1 = 20 \text{ kN} = 20 \times 10^3 \text{ N}$

$L_1 = 3.5 \text{ m} = 3500 \text{ mm}$

$P_2 = 30 \text{ kN} = 30 \times 10^3 \text{ N}$

$L_2 = 4 \text{ m} = 4000 \text{ mm}$

$P_3 = 20 \text{ kN} = 20 \times 10^3 \text{ N}$

$L_3 = 7.5 \text{ m} = 7500 \text{ mm}$

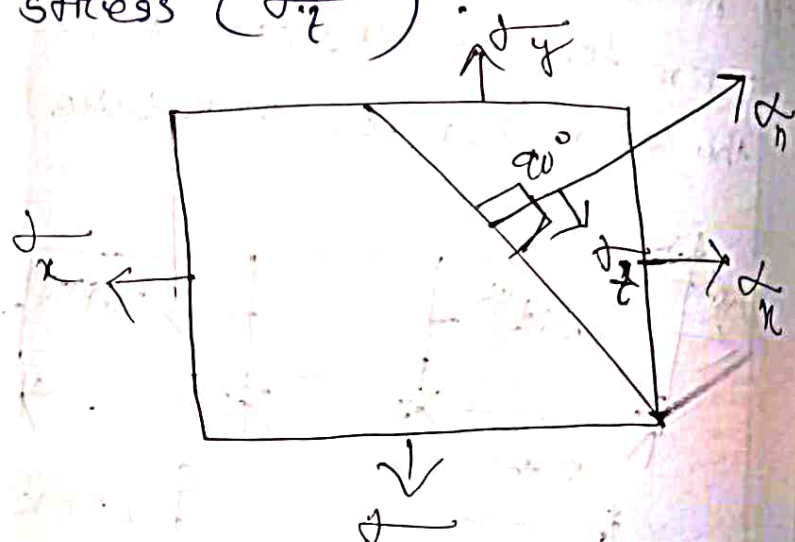
$$\Delta L = \frac{1}{AE} (P_1 L_1 + P_2 L_2 + P_3 L_3)$$

$$= \frac{1}{800 \times 100 \times 10^3} (20 \times 10^3 \times 3500 + 30 \times 10^3 \times 4000 + 20 \times 10^3 \times 7500)$$

$$= 4.63 \text{ mm Ans}$$

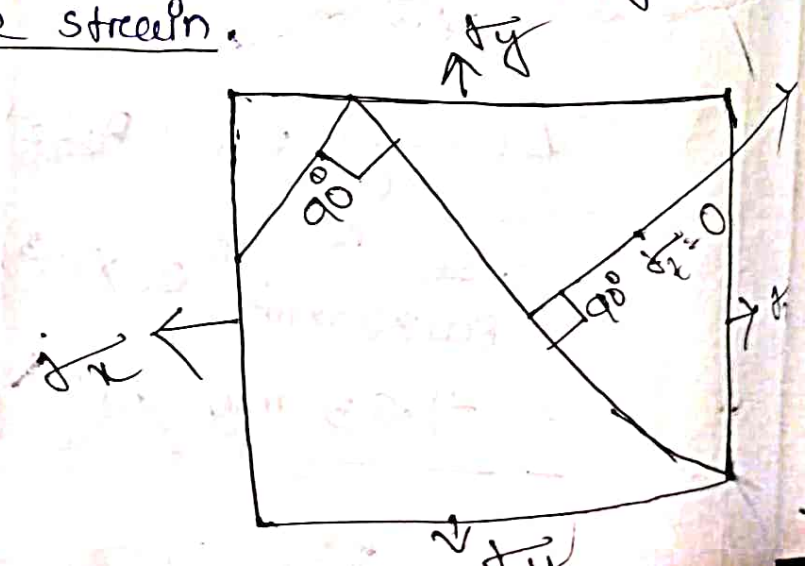
Complex stress & strain

→ When a material is subjected to composite stresses system in any plane there will arise two types of stresses at right angle to each other. The stresses at 90° to the plane is known as Normal stress (σ_n) and the stress parallel to the plane is known as tangential stress (σ_t).



→ At a particular plane there will be only normal stresses & no tangential stress such a particular plane is known as principle plane.

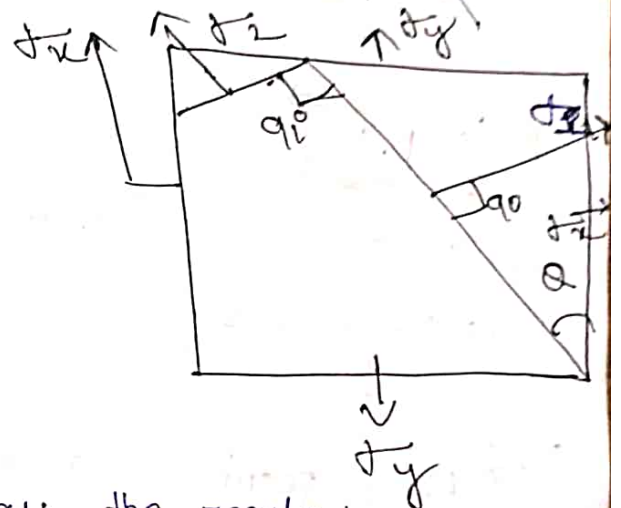
→ The Normal stress acting on this plane is known as principle stress and the corresponding strain is known as principle strain.



Dt - 1.11.2022

In a actual practice there will be two sets of such planes at right angle to each other, where the tangential or shear will be '0'

They are known as ~~measure~~ ^{major} principle stress & minor principle stress. plane. the normal acting on this plane is known as major principle stress (σ_1) & minor principle stress (σ_2).



OBLIQUITY $\rightarrow$ The angle made by the resultant stress with the normal of the obliquity plane is known as Obliquity.

It is denoted by ' ϕ '.

$$\tan \phi = \frac{\sigma_t}{\sigma_n}$$

$$\phi = \tan^{-1} \left(\frac{\sigma_t}{\sigma_n} \right)$$

Some Important Formula:-

$$\text{Normal stress } (\sigma_n) = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + q \sin 2\theta$$

Tangential stress (σ_t)

$$= \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta - q \cos 2\theta$$

$$\rightarrow \text{Resultant stress} = \sigma_R = \sqrt{(\sigma_n)^2 + (\sigma_t)^2}$$

$$\rightarrow \text{OBLIQUITY} = \phi = \tan^{-1} \left(\frac{\sigma_t}{\sigma_n} \right)$$

$\rightarrow$ Major principle plane & minor principle plane

$$(\text{stress } \sigma_1 \& \sigma_2) = \left(\frac{\sigma_x + \sigma_y}{2} \right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + q^2}$$

$\downarrow$
 Shear stress

★ N.B ★

Positive sine for major principle stress or plane.

Negative sine for minor principle stress or plane.

Q1// At a point in a strain material the principle stresses are 100 N/mm^2 (tensile) & 60 N/mm^2 (compressive). Determine the normal stress, tangential stress & Resultant on a plane inclined at 30° to the major principle stress and also determine the obliquity!

Given $\sigma_x = 100 \text{ N/mm}^2$

$\sigma_y = 60 \text{ N/mm}^2 = -60 \text{ N/mm}^2$

$\theta = 30^\circ$

Normal stress =

$$= \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + q \sin 2\theta$$

$$= \left(\frac{100 + (-60)}{2} \right) + \left(\frac{100 - (-60)}{2} \right) \cos 2 \times 30^\circ +$$

$$= 6.10 \text{ N/mm}^2$$

→ tangential stress

$$\begin{aligned} & \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta - \tau \cos 2\theta \\ &= \left(\frac{100 - (-60)}{2} \right) \sin 2 \times 50 \\ &= 78.78 \text{ N/mm}^2 \end{aligned}$$

→ Major principle stress
Resultant stress

$$\begin{aligned} \sigma_R &= \sqrt{(\sigma_n)^2 + (\sigma_t)^2} \\ &= \sqrt{(6.10)^2 + (78.78)^2} \\ &= 79.01 \text{ N/mm}^2 \end{aligned}$$

→ Obliquity (ϕ) = $\tan^{-1} \left(\frac{\sigma_t}{\sigma_n} \right)$

$$\begin{aligned} &= 85.57 \\ &= 85^\circ 34' 20.55'' \end{aligned}$$

Q2/ The principle tensile at point stresses occur on two perpendicular plane are 800 kg/cm^2 & 400 kg/cm^2
find the normal stress, tangential stress & resultant stress & its obliquity on a plane on a 20° with a major principle plane?

Q11

Given

$$\sigma_x = 800 \text{ kg/cm}^2$$

$$\sigma_y = 400 \text{ kg/cm}^2$$

$$\theta = 20^\circ$$

Normal stress (σ_n)

$$\begin{aligned}\sigma_n &= \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau \sin 2\theta \\ &= \left(\frac{800 + 400}{2} \right) + \left(\frac{800 - 400}{2} \right) \cos 2 \times 20 \\ &= 753.2 \text{ N/cm}^2\end{aligned}$$

tanjensia stress (τ)

$$\begin{aligned}& \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta - \tau \cos 2\theta \\ &= \left(\frac{800 - 400}{2} \right) \sin 2 \times 20^\circ \\ &= 128.8 \text{ N/cm}^2\end{aligned}$$

Resultant stress (σ_R)

$$\begin{aligned}\sigma_R &= \sqrt{(\sigma_n)^2 + (\tau)^2} \\ &= \sqrt{(753.2)^2 + (128.8)^2} \\ &= 768.08 \text{ N/cm}^2\end{aligned}$$

obliquity:- $\phi = \tan^{-1} \left(\frac{\tau}{\sigma_n} \right)$

$$= \tan^{-1} \left(\frac{128.8}{753.2} \right)$$

$$= 9.91$$

$$= 9^\circ 40' 40''$$

$$= 9^\circ 40' 40''$$

Q3// A point
Formula

6. position of principle plane $\tan 2\alpha = \frac{2q}{\sigma_x - \sigma_y}$
or $\alpha = \frac{1}{2} \tan^{-1} \left(\frac{2q}{\sigma_x - \sigma_y} \right)$

7. position & direction & magnitude of maximum shear stress

$\tau_{max} = \left(\frac{\sigma_1 - \sigma_2}{2} \right)$
or $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + q^2}$

Q4) A certain point in a strain material the intensity of the normal stress occurs two planes are 800 kg/cm^2 & 320 kg/cm^2 both tensile. noted the principle plane, & evaluate the principle stress & also calculate the maximum intensity of shear stress & specify its plane. there is a shear of 320 kg/cm^2 acting its plane.

Given $\sigma_x = 800 \text{ kg/cm}^2$
 $\sigma_y = 320 \text{ kg/cm}^2$
 $q = 320 \text{ kg/cm}^2$

Principle plane
 $\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + q^2}$
 $= \left(\frac{800 + 320}{2} \right) + \sqrt{\left(\frac{800 - 320}{2} \right)^2 + 320^2}$
 $= 960 \text{ kg/cm}^2$

$$\sigma_2 = \left(\frac{800 + 320}{2} \right) - \sqrt{\left(\frac{800 - 320}{2} \right)^2 + 320^2}$$

$$= 160 \text{ kg/cm}^2$$

Position of principle stress plane

$$\tan 2\alpha = \frac{2q}{\sigma_x - \sigma_y}$$

$$= \tan 2\alpha = \frac{2 \times 320}{800 - 320}$$

$$= \alpha =$$

$$\alpha = \frac{1}{2} \tan^{-1} \left(\frac{2q}{\sigma_x - \sigma_y} \right)$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{2 \times 320}{800 - 320} \right)$$

$$= 26.57 = 26^\circ 33'$$

$$\therefore \alpha = \alpha_1$$

$$\alpha_2 = \alpha_1 + 90^\circ = 116^\circ 33'$$

$$\rightarrow q_{\max} = \left(\frac{\sigma_1 - \sigma_2}{2} \right)$$

$$= \left(\frac{960 - 160}{2} \right) = 400 \text{ kg/cm}^2$$

$$\begin{cases} \alpha = \alpha_1 \\ \alpha_2 = \alpha_1 + 90^\circ \\ \beta_1 = \alpha_2 + 45^\circ \end{cases}$$

→ position of shear stress

$$\beta_1 = \alpha_1 + 45^\circ$$

$$\beta_2 = \alpha_1 + 135^\circ$$

$$\beta_1 = 26.57 + 45^\circ =$$

$$\beta_2 = 26.57 + 135^\circ =$$

Q. A rectangular block of material is subjected to a stress of 1100 kg/cm^2 from one plane and tensile stress of 470 kg/cm^2 on a plane at right angles to former which shear stress of 630 kg/cm^2 on the same plane.

Find i) The direction of the principal plane.

ii) The magnitude of the principal stress.

iii) The magnitude of the greater shear stress.

Given

$$\sigma_x = 1100 \text{ kg/cm}^2$$

$$\sigma_y = 470 \text{ kg/cm}^2$$

$$\tau = 630$$

major principal plane (σ_1)

$$= \left(\frac{\sigma_x + \sigma_y}{2} \right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau^2}$$

$$= \left(\frac{1100 + 470}{2} \right) + \sqrt{\left(\frac{1100 - 470}{2} \right)^2 + 630^2}$$

$$= 1489.36$$

minor principal plane (σ_2)

$$\left(\frac{\sigma_x + \sigma_y}{2} \right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau^2}$$

$$\left(\frac{1100 + 470}{2} \right) - \sqrt{\left(\frac{1100 - 470}{2} \right)^2 + 630^2}$$

$$= 80.64$$

i) position of principal plane

$$\alpha = \frac{1}{2} \tan^{-1} \left(\frac{2\tau}{\sigma_x - \sigma_y} \right)$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{2 \times 630}{1100 - 470} \right)$$

$$= 31.71$$

$$= 31^\circ 43'$$

ii) principle of share stress

$$\beta_1 = \alpha + 45^\circ$$

$$= 31^\circ 43' + 45^\circ$$

$$= 76^\circ 43'$$

$$\beta_2 = \alpha + 135^\circ$$

$$= 31^\circ 43' + 135^\circ$$

$$= 166^\circ 43'$$

iii) maximum of share stress

$$\tau_{\max} = \left(\frac{\sigma_1 - \sigma_2}{2} \right)$$

$$= \left(\frac{1489.36 - 80.64}{2} \right)$$

$$= 704.36 \text{ kg/cm}^2$$

Q2/ A point in a strained material is subjected to two mutually perpendicular tensile stresses of 200 MPa and 100 MPa. Determine the intensities of normal, shear and resultant stresses on a plane inclined at 30° with the axis of minor tensile stress.

Given $\sigma_x = 200 \text{ MPa}$
 $\sigma_y = 100 \text{ MPa}$
 $\theta = 30^\circ$

(1) σ_n (normal stress)

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$= \frac{200 + 100}{2} + \frac{200 - 100}{2} (\cos 2 \times 30^\circ) = 175 \text{ MPa}$$

Shear stress $q_{\max} = \left(\frac{\sigma_x - \sigma_y}{2} \right)$ $= \left(\frac{200 - 100}{2} \right)$	Resultant stress $\sigma_R = \sqrt{(\sigma_n)^2 + (q)^2}$
--	--

Resultant stress :-

$$\sigma_R = \sqrt{(\sigma_n)^2 + (q)^2}$$

$$= \sqrt{(175)^2 + (43.30)^2}$$

$$= 180.27$$

(2) Tangential stress

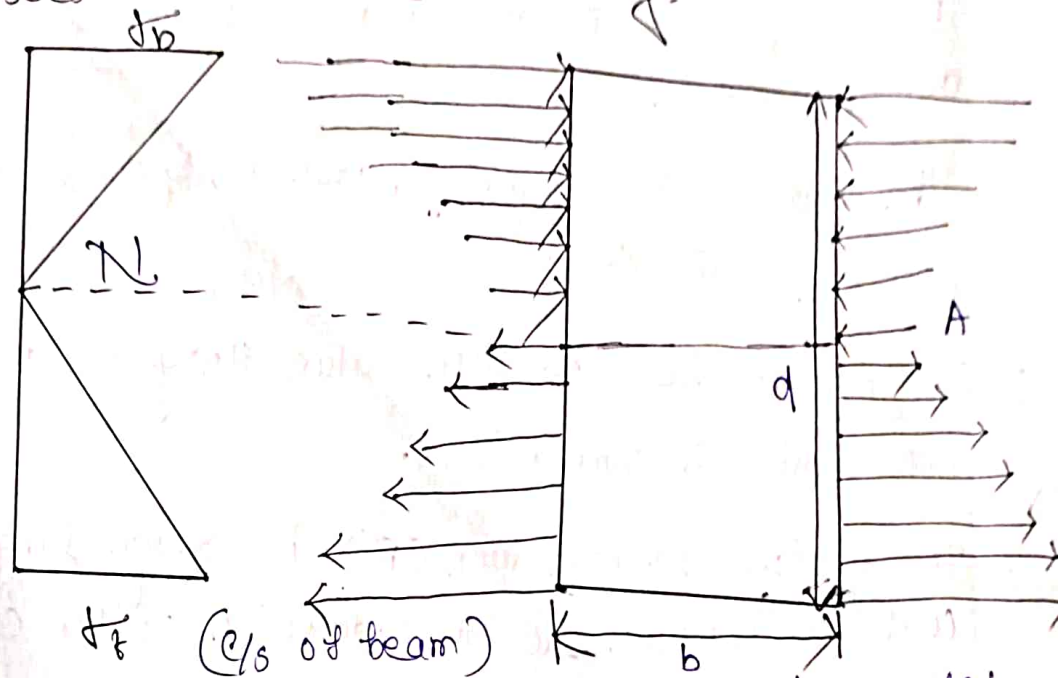
$$= \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta - q \cos 2\theta$$

$$= \left(\frac{200 - 100}{2} \right) \sin 2 \times 30^\circ$$

$$= 43.30 \text{ MPa}$$

Stresses In Beams and shafts

→ stresses in beams due to bending:-



→ when the beam is bent they exist a layer which neither get compressed nor elongated.

→ This layer is known as neutral layer & stress of the neutral surface on the cross section is the neutral axis.

→ The neutral axis is that axis where the bending stress is '0'.

→ Above the neutral axis is known as compressive zone & below the neutral axis is called as tensile zone.

★ Simple bending or Pure bending:-

→ when every cross-section set up full resistance to bending moment. The resistance offered by the internal stresses to the bending is called bending stress & the relevant theory is called theory of simple bending.

- Assumption made in simple bending :→
- The cross section of the beam which is plane & normal before bending remains plane & normal after bending.
- The material is homogenous, isotropic & obeys Hooke's law within elastic limit.
- The material has the same value of the young's modulus both in tension & compression.
- The radius of curvature (R) is very large when compared to the dimension of the cross section of the beam.
- The beam is subjected to pure bending therefore it bends into an arc of a circle.
- The beam is in equilibrium that is there is no resultant pull or push in the beam section.

→

Q3// Chapter 2

The stresses at point of a machine component are 150 MPa & 50 MPa both tensile. Find the intensities of normal, shear and resultant stresses on a plane inclined at an angle of 55° with the axis of the maximum shear stress in the component?

Given

$$\sigma_x = 150 \text{ MPa}$$

$$\sigma_y = 50 \text{ MPa}$$

$$\theta = 55^\circ$$

$$\begin{aligned} \textcircled{1} \quad \sigma_n &= \left(\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta \right) \\ &= \left(\frac{150 + 50}{2} + \frac{150 - 50}{2} \cos 2 \times 55^\circ \right) \\ &= 82.89 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \tau_z &= \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta \\ &= \left(\frac{150 - 50}{2} \right) \sin 2 \times 55^\circ \\ &= 46.98 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \text{Resultant stress} &\dots \\ \sigma_R &= \sqrt{(\sigma_n)^2 + (\tau_z)^2} \\ &= \sqrt{(82.89)^2 + (46.98)^2} \\ &= 95.27 \text{ MPa} \end{aligned}$$

D-04.11.2022



Bending Equation:-

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

where M is bending (movement) in

I - Moment of Inertia in $\text{mm}^4, \text{cm}^4, \text{m}^4$

σ - bending stress in N/mm^2 or kg/cm^2
or KN/m^2

moment = force $\times$ perpendicular distance

y = maximum centrodal distance from the neutral axis in mm or cm

E = youngs modulus in N/mm^2 or kg/cm^2 or KN/m^2

R = Radius of curvature in cm or mm or m.



Section modulus :-

equation $\frac{M}{I} = \frac{\sigma}{y}$

$$\Rightarrow M = \frac{\sigma}{y} \times I$$

or

$$\Rightarrow M = \sigma \times Z \left(\because \frac{I}{y} = Z \right)$$

→ The Quantity $\frac{I}{y}$ is called section modulus or
It is the ratio between moment of Inertia to the maximum centrodal distance from the neutral axis is know as section modulus.

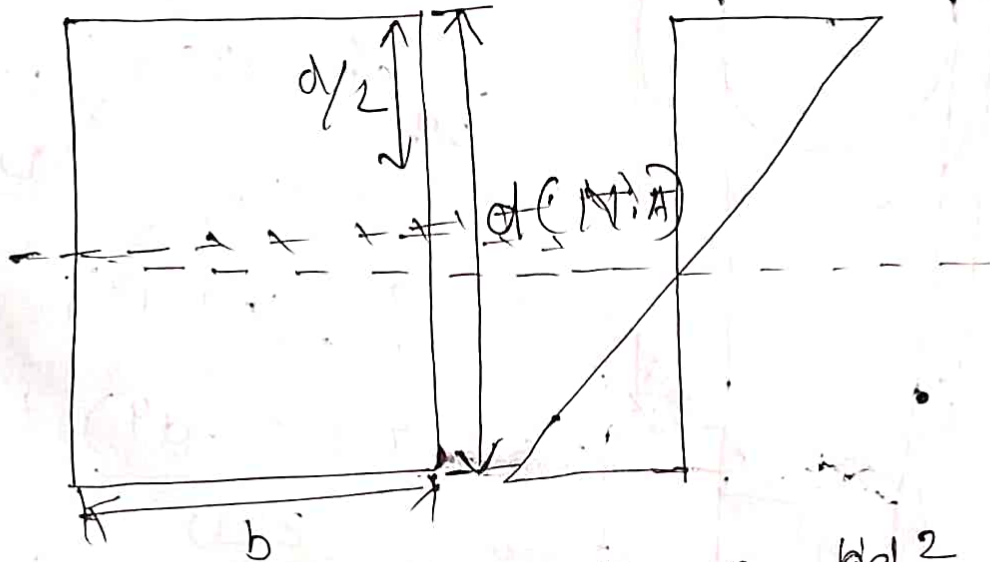
→ It is denoted by 'Z'

→ It's unit is cm^3, m^3 and mm^3 .

→ To measure the strength of cross section of the bending.

→ Section modulus for different section:-

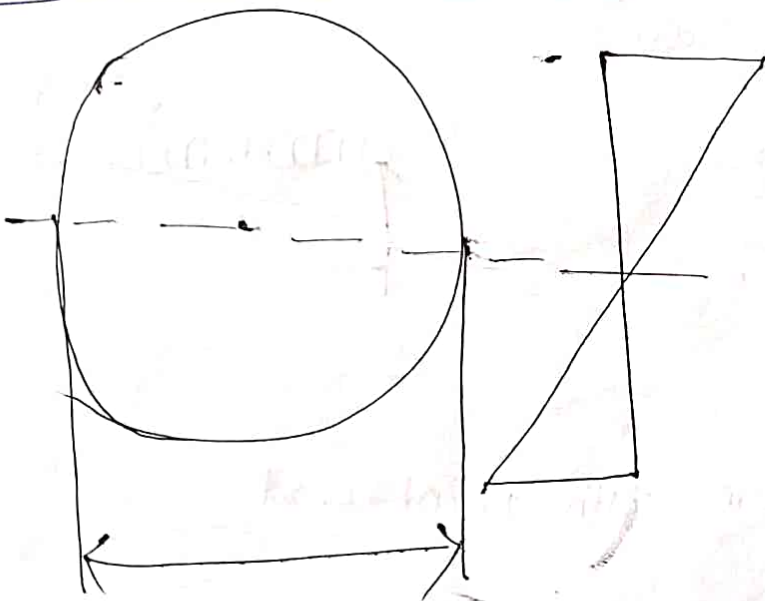
→ 1st section is rectangle section.



$$Z = \frac{I}{Y} = \frac{\frac{bd^3}{12}}{d/2} = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$$

$$\therefore Z = \frac{bd^2}{6}$$

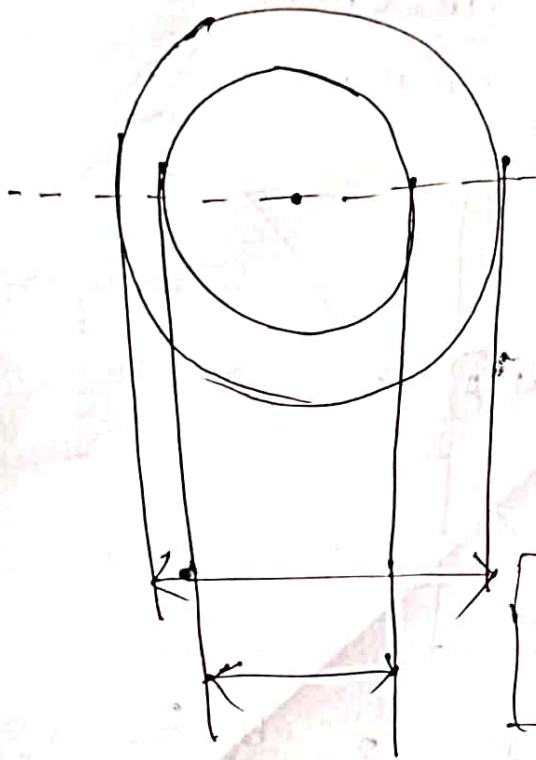
2. Circular section



$$Z = \frac{I}{Y} = \frac{\frac{\pi}{64} \times d^4}{d/2} = \frac{\pi d^4}{64} \times \frac{2}{d}$$

$$Z = \frac{\pi d^3}{32}$$

3. Hollow Circular Section:-



$$Z = \frac{I}{Y} = \frac{\frac{\pi}{64} (D^4 - d^4)}{\frac{D}{2}}$$

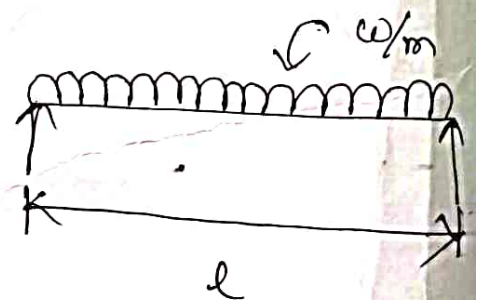
$$= \frac{\pi (D^4 - d^4)}{32D}$$

$$Z = \frac{\pi (D^4 - d^4)}{32D}$$

DT - 05.11.2022

→ Maximum bending moment

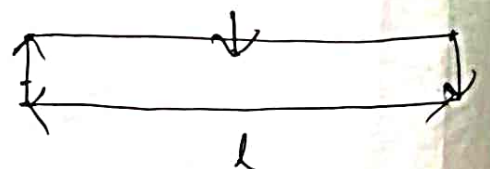
For simply supported beam with UDL
(uniformly distributed load).



$$M_{max} = \frac{wL^2}{8}$$

→ Simply supported beam with point load

$$M_{max} = \frac{WL}{4}$$

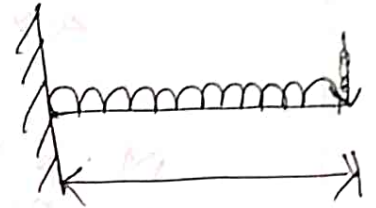
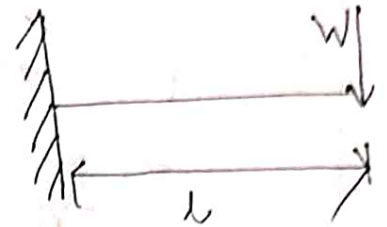


→ For cantilever beam

$$M_{\max} = WL$$

→ For cantilever beam with UDL

$$M_{\max} = \frac{WL^2}{2}$$



Q1// A cast iron cantilever of ~~length~~ 1.5 m fails when a load of ~~9000~~ 1920 N is applied at the free end. Determine the stress at failure if the section of the cantilever is 40 x 60 mm²?

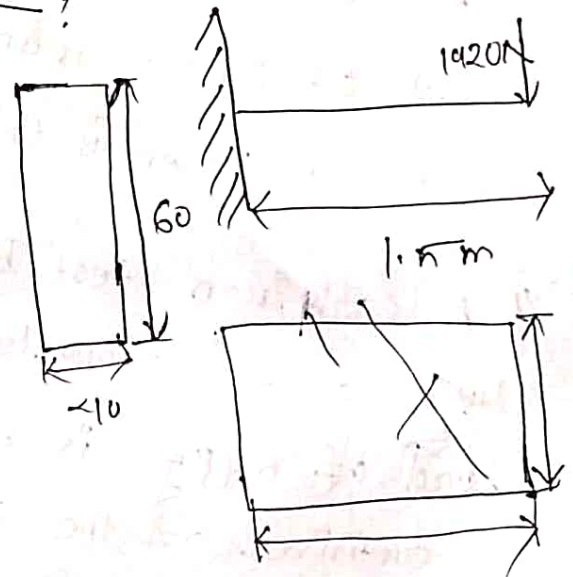
Given

$$L = 1.5 \text{ m} = 1500$$

$$W = 1920 \text{ N}$$

$$B = 40$$

$$D = 60$$



We know

the bending eqⁿ $\frac{M}{I} = \frac{\sigma}{y}$

Therefore $\sigma = \frac{M}{I} xy$

for cantilever beam $M_{\max} = WL = 0$
 $1500 \times 1920 = 288 \times 10^4 \text{ N-mm}$

$$y = \frac{d}{2} = \frac{60}{2} = 30 \text{ mm}$$

$$I = \frac{bd^3}{12} = \frac{40 \times 60^3}{12} = 72 \times 10^4 \text{ mm}^4$$

$$\sigma = \frac{M}{I} xy = \frac{288 \times 10^4}{72 \times 10^4} \times 30 = 120 \text{ N/mm}^2$$

OR

$$M = f \times Z$$

$$Z = \frac{bd^2}{6}$$

$$= \frac{40 \times 60^2}{6} = 24 \times 10^4 \text{ mm}^3$$

$$M = f \times Z$$

$$f = \frac{M}{Z}$$

$$= \frac{288 \times 10^4}{24 \times 10^4} = 120 \text{ N/mm}^2$$

Q2/ A cast iron cantilever of length 1.5m fails when a load of 2kN is applied at free end determine the stress at failure if the cantilever is 50x70mm in section?

Q3/ A cast iron test beam 20cm x 20cm in section and 1m long is supported at the ends fails when the central load of 64kg is applied what is the UDL will break a cantilever of the same material, 5cm wide & 10cm deep & 2m long?

Given

$$L = 1\text{m} = 1000\text{cm}$$

$$W = 64\text{kg}$$

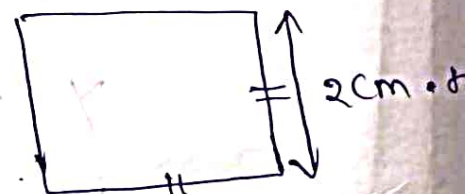
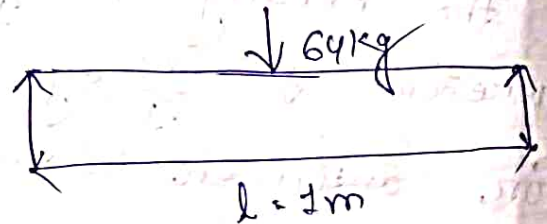
$$B = 20\text{cm}$$

$$D = 5\text{cm}$$

1st case

$$M = f \times Z$$

$$f = \frac{M}{Z}$$



maximum bending moment for cantilever beam

$$M_{\text{max}} = \frac{WL}{4} = \frac{64 \times 1000}{2} = 3200 \text{ N/cm}$$

$$Z = \frac{bd^3}{6} = 1.33 \text{ cm}^3$$

$$f = \frac{1600}{1.33} = 1200.3 \text{ kg}$$

②. second case

Given $L = 2 \text{ m}$

$$d = 10 \text{ cm}$$

$$b = 5 \text{ cm}$$

$$Z = \frac{5 \times 10^3}{6} = 83.33$$

$$M = f \times Z$$

$$= 1200.3 \times 83.33$$

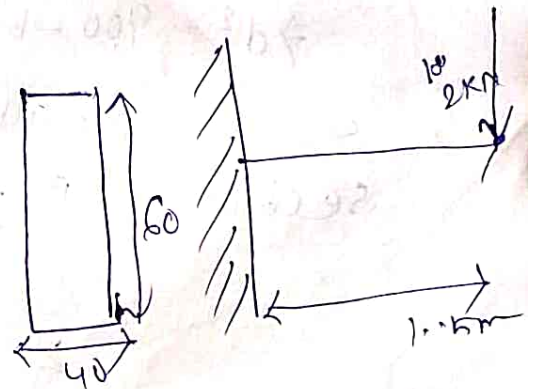
$$= 10 \times 10^4$$

$$M_{\text{max}} = \frac{WL^2}{2}$$

$$W = \frac{M^2}{L^2}$$

$$= 5 \text{ kg}$$

$$\text{UDL} = 5 \text{ kg/cm}$$



2// Given $L = 1.5 \text{ m} = 1500 \text{ mm}$

$$W = 2 \text{ kN} = 2000 \text{ N}$$

$$B = 50 \text{ mm}$$

$$D = 70 \text{ mm}$$

$$Z = \frac{BD^3}{6} = \frac{50 \times 70^3}{6}$$

$$= 40833.33 \text{ mm}^3$$

$$M_{\text{max}} = W \times L$$

$$= 2000 \times 1500$$

$$= 3000000 \text{ Nmm}$$

$$M = f \times Z$$

$$f = \frac{M}{Z} = \frac{3000000}{40833.33} = 73.469 \text{ N/mm}^2$$

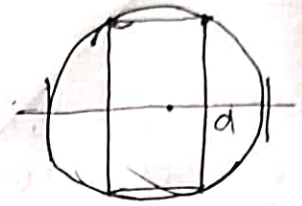
Dt: 9.11.2022

→ To calculate the width & depth of the strongest beam of a rectangular section that can be cut from a cylindrical wood whose diameter is 300 mm?

Given:

$$D = 300 \text{ mm}$$

Let, the strongest section be of a rectangle of width b in cm & depth d in cm.



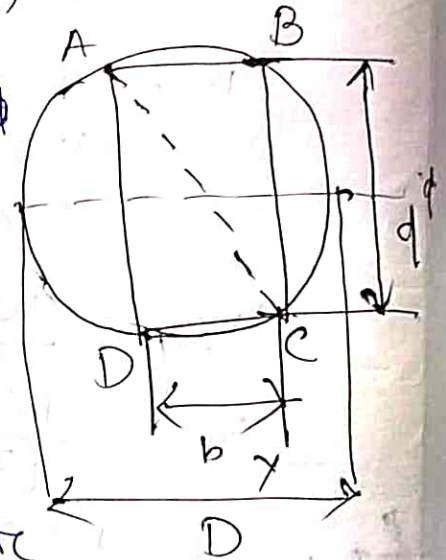
⇒ From the figure in triangle ACD

$$b^2 + d^2 = D^2$$

$$b^2 + d^2 = 300^2$$

$$b^2 + d^2 = 900$$

$$\Rightarrow d^2 = 900 - b^2 \quad \text{--- (1)}$$



section modulus for rectangular section. $\therefore Z = \frac{bd^2}{6}$

$$Z = \frac{b \times (900 - b^2)}{6}$$

$$Z = \frac{900b - b^3}{6} = \frac{900b}{6} - \frac{b^3}{6}$$

for the strongest section $b = \frac{dZ}{db} = 0$

$$\Rightarrow \frac{d}{db} \left(\frac{900b}{6} - \frac{b^3}{6} \right) = 0$$

$$\Rightarrow \frac{900}{6} \left(\frac{d}{db} b \right) - \frac{1}{6} \left(\frac{d}{db} b^3 \right) = 0$$

$$\Rightarrow 150 \times 1 - \frac{1}{6} (3 \times b^2) = 0$$

$$= 150 - \frac{b^2}{2} = 0$$

$$b^2 = 150 \times 2 = 300$$

$$b = \sqrt{300} = 10\sqrt{3} = 17.321 \approx 18 \text{ cm}$$

put the value of 'b' in eqn (1)

$$d^2 = 900 - b^2 \quad \text{--- (1)}$$

$$d = \sqrt{900 - 300}$$

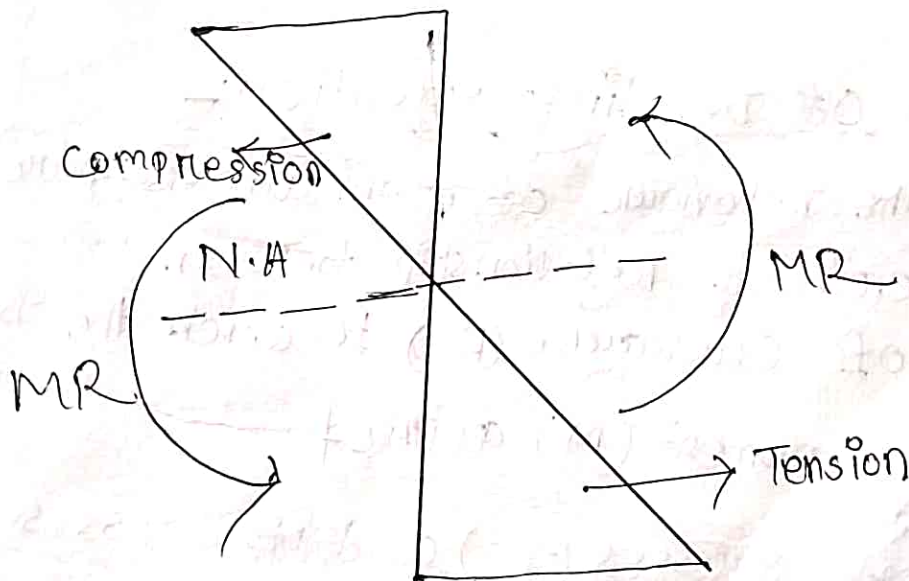
$$= \sqrt{600} = 24.495 \text{ cm} \approx 25 \text{ cm}$$

Stresses in beams due to bending 3rd chapter

Moment of resistance :-

The bending stresses induced in the cross section of a beam constitute a couple. Moment of this couple is known as moment of resistance which resists the external bending moment at that section.

It is defined the resistance offered by the cross section of the beam for the external bending moment.



Flexural rigidity:-

- Flexural rigidity is defined as the force couple required to bend a non-rigid structure in one unit of curvature or it can be defined as the resistance offered by a structure while undergoing bending.
- In a beam or rod, flexural rigidity (defined as EI) varies along the length as a function of x shown in the following equation,

$$EI \frac{d^2y}{dx^2} = \int_0^x M(x) dx + C_1$$

where

E is the young's modulus (in Pa)

I is the moment of area (in m^4)

y is the transverse displacement of the beam,

$M(x)$ is the bending moment at x .

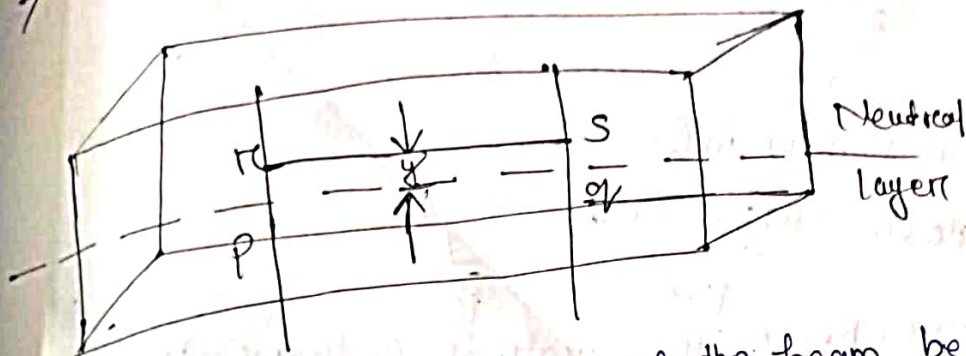
- Flexural rigidity has SI units of $Pa \cdot m^4$ which is also equal to $N \cdot m^2$.

Derivation of bending equation:-

- To study the behaviour of beam under pure bending let us derive a relationship between
 - Radius of curvature (R) to which the beam bends.
 - Bending moment (M) applied
 - Bending stresses (σ) and its cross sectional dimension property (I).
- The equation which connects these quantities is known as the bending equation.

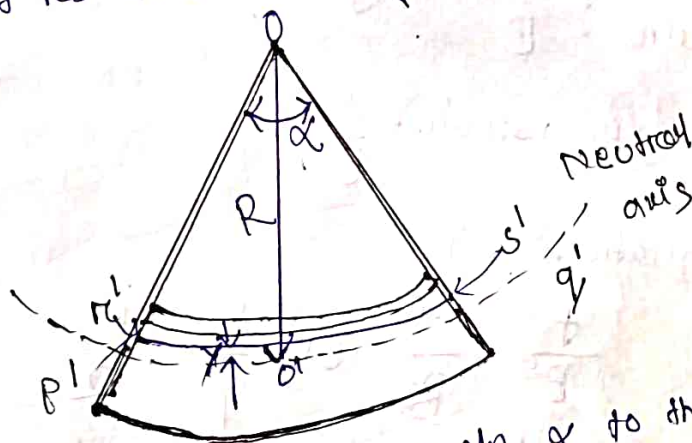
→ Expression for bending stress:-

→ Let us analyze the beam subjected to pure bending.



→ Length of neutral layer of the beam before bending is pq .
 → Let us take another layer of length $rs = pq$ at a distance y from pq .

→ Beam bends to a radius of curvature $R = OO' = OP'$.
 → After bending, neutral layer occupies $p'q'$, since $p'q'$ lies on the N.A its length does not change. $pq = p'q'$.
 → Length of rs after bending becomes $r's'$.



→ $p'q'$ and $r's'$ subtends an angle α to the centre of curvature, O

$$p'q' = R\alpha$$

$$r's' = (R-y)\alpha$$

→ Before bending, parallel layers pq and rs have equal length
 $pq = rs$

→ Since there is no change in length of the neutral layer
 (No strain), there is no strain.

so $p'p' = pq$
 But $(rs = pq = p'q')$

$$\begin{aligned}\text{Strain in } r_s &= \frac{\text{Change in length}}{\text{Original length}} \\ &= \frac{r_s \alpha - r_s}{r_s}\end{aligned}$$

Substituting equation (1) in (2)

$$\text{strain} = \frac{(R+y)\alpha - R\alpha}{R\alpha} = \frac{y}{R}$$

- As R is constant, hence the strain in a layer is proportional to its distance from the neutral axis
- The above equation shows the variation of strain along the depth of the beam.
- The variation of strain is linear
- Stress variation
- Let us take bending stress along, $r_s = \sigma$
- Young's modulus = E
- By Hooke's law, strain = $\frac{\sigma}{E}$

From equation (3)

$$\frac{\sigma}{E} = \frac{y}{R} \quad \text{or} \quad \frac{\sigma}{y} = \frac{E}{R}$$

$$\text{Strain} = \frac{y}{R}$$

$$\sigma = \frac{E}{R} \times y$$

- Since E and R are constant, therefore stress in any layer is directly proportional to the distance of the layer from the neutral layer.

→ Stresses in shafts due to torsion

concept of torsion:- The strain eqn or stiffness eqn or torsion eqn is $\frac{T}{J} = \frac{q}{R} = \frac{C\theta}{L}$

where T = Torque in N-mm

J = Polar moment of Inertia in mm

q = shear stress N/mm²

R = Radius of circular shaft in mm

'C' or 'G' modulus rigidity

C = is a constant is also known as torsional rigidity for the stiffness eqn.

θ = angle in radian

L = Length of the shaft in mm.

→ Assumption for pure torsion:-

→ The Theory of pure torsion based on the following assumption.

→ The material of the shaft is uniform through out.

→ The angle of twist is uniform

→ The shaft is of uniform circular cross-section through out.

→ The cross-section of the shaft which are plane before twist & remain plane after twist.

→ Remain straight with their magnitude after the twist.

→ Polar moment of Inertia:

→ We know the stiffness eqn $\frac{T}{J} = \frac{q}{R} = \frac{C\theta}{L}$

$\frac{q}{R} = \frac{C\theta}{L}$ & we know

$$T = Z_p \times q$$

$$\therefore Z_p = \frac{\pi D^3}{16}$$

$$= \frac{\pi D^3}{16} \times q = q = \frac{T \times 16}{\pi D^3}$$

→ The value of q put in stiffness eqⁿ

$$\frac{q}{r} = \frac{C\theta}{L}$$

$$\Rightarrow \frac{T \times 16}{\pi D^3 \times \pi} = \frac{C\theta}{L}$$

$$\Rightarrow \frac{T \times 16 \times 2}{\pi \times D^3 \times D} = \frac{C\theta}{L}$$

$$\Rightarrow \frac{T}{\frac{\pi D^4}{32}} = \frac{C\theta}{L}$$

$$\Rightarrow \frac{T}{J} = \frac{C\theta}{L}$$

$$\boxed{J = \frac{\pi D^4}{32}}$$

for hollow section

$$\boxed{J = \frac{\pi (D^4 - d^4)}{32}}$$

where J is called polar m.I

⇒ Torsional rigidity:- In stiffness eqⁿ $\frac{T}{J} = \frac{q}{r} = \frac{C\theta}{L}$

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$C = T \times \frac{L}{J\theta} \text{ or}$$

Angle of twist

$$\boxed{\theta = \frac{T \times L}{J \times C}}$$

Q1// Calculate the minimum of diameter of solid shaft if it is to transmit a torque of 150000 N.m without the max shear stress exceeding 45 mpa.

$$\text{Given } T = 150,000 \text{ N.m} = 150,000 \times 10^3 \text{ N.mm}$$

$$q = 45 \text{ mpa} = 45 \text{ N/mm}^2$$

$$\frac{T}{J} = \frac{q}{r}$$

We know the stiffness eqⁿ. $\frac{T}{J} = \frac{q}{r}$

$$\rightarrow J = \frac{\pi D^4}{32}$$

$$\rightarrow r = \frac{D}{2}$$

$$\rightarrow \frac{1501000 \times 10^3}{\frac{\pi D^4}{32}} = \frac{4\pi}{\frac{D}{2}}$$

$$\rightarrow \frac{1501000 \times 10^3 \times 32}{\pi D^4} = \frac{90}{D}$$

$$\rightarrow D^3 = \frac{1501000 \times 10^3 \times 32}{\pi \times 90}$$

$$\rightarrow D = \sqrt[3]{16976527.26}$$

$$= 257 \text{ mm}$$

2// A hollow shaft of external dia 10cm & thickness 1cm is subjected to a torque the max shear 800kg/cm² find the magnitude of safe torque.

Given $D = 10 \text{ cm}$

$t = 1 \text{ cm}$

$q = 800 \text{ kg/cm}^2$

$d = 10 - (1 + 1) = 8 \text{ cm}$

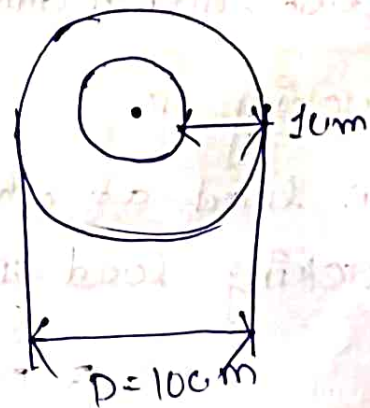
$T = ?$

$$\frac{T}{J} = \frac{q}{r}$$

$$\rightarrow J = \frac{\pi}{32} \times (D^4 - d^4)$$

$$= \frac{\pi}{32} \times (10^4 - 8^4) = 579.62$$

$$r = \frac{D}{2} = \frac{10}{2} = 5 \text{ cm}$$



Column → These are the vertical members used in building frame
Struts! — It is a compression member of the structure
subjected to an axial compressive force is known as
struts.

Classification of columns

→ Columns are classified into categories :-

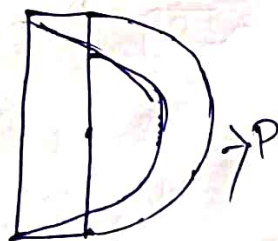
- 1) Long column
- 2) Short column

→ Long column (effective length)
Least lateral dimension > 12

→ Short column
 $\frac{\text{effective length}}{\text{Least lateral dimension}} < 12$

→ Buckling or crippling load

→ The load at which the member just buckles is called
Buckling load or critical load or crippling load.



→ It is denoted by 'P'

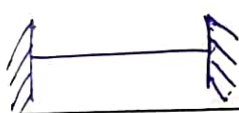
$$P = \frac{\pi^2 EI}{L^2}$$

where p = Buckling load.

E = Modulus of Elasticity

I = Moment of Inertia

L = effective length

End conditions of column	Crippling load	Relation bet ⁿ effective length & Actual length
① Both end hinged 	$\frac{\pi^2 EI}{L^2} = \frac{\pi^2 EI}{L^2}$	$L = l$
② One end fixed and other free 	$\frac{\pi^2 EI}{4l^2} = \frac{\pi^2 EI}{L^2}$	$L = 2l$
③ Both end fixed 	$\frac{4\pi^2 EI}{L^2} = \frac{\pi^2 EI}{L^2}$	$L = \frac{l}{2}$
④ One end fixed and one end hinged 	$\frac{2\pi^2 EI}{L^2} = \frac{\pi^2 EI}{L^2}$	$L = \frac{l}{\sqrt{2}}$

Slenderness Ratio : L = effective length
 l = Actual length

It is define as the ratio effective length to the ratio of gyration of the section.

Mathematically slenderness ratio $= \frac{\text{effective length (L)}}{\text{gyration}}$

$$SR = \frac{L}{K}$$

~~Assum~~ Made of

Assumption Load of Euler's theory:-

- The column is initially perfect straight axially loaded
- The section of column is uniform.
- The column material is perfectly elastic, homogeneous and isotropic and thus obeys Hooke's law.
- The length of column is very large as compared to lateral dimension.
- The shortening of column is ignorable.
- The column will fail by buckling alone

Problem-1

A steel rod 3m long π cm in diameter is used as struts with both end hinged. Determine the crippling load take E is $2 \times 10^5 \text{ N/mm}^2$

$$\text{Given } L = 3\text{m} = 3000\text{mm}$$

$$D = \pi \text{ cm} = \pi \text{ mm}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$P = ?$$

$$P = \frac{\pi^2 EI}{L^2}$$

$$\text{We know } I = \frac{\pi}{64} \times d^4$$

$$= \frac{\pi}{64} \times \pi^4$$

$$= 306.80 \times 10^3 \text{ mm}^4$$

$$P = \frac{\pi^2 \times 2 \times 10^5 \times 306.80 \times 10^3}{3000^2} = 67.29 \times 10^3 \text{ N } \underline{\underline{\text{Ans}}}$$

P

Problem :- 2

→ A hollow alloy tube 5m long with external and internal dia 40mm & 25mm respectively was found to extend 6.4mm under a tensile load of 60kN, find the buckling load for the tube when used as a column with both ends are pinned, also find safe load on the tube and factor of safety 4.

Given

$$L = 5\text{m} = 5000\text{mm} = \frac{5000}{2} = 2500\text{mm}$$

$$D = 40\text{mm}$$

$$d = 25\text{mm}$$

$$P = 60\text{kN} = 60000\text{N}$$

$$\Delta L = 6.4\text{mm}$$

$$P = ?$$

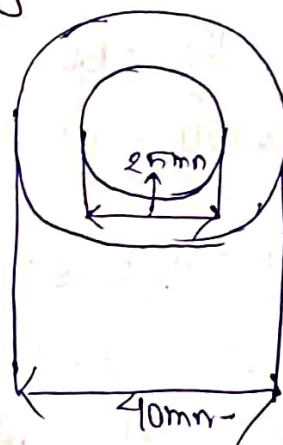
Condition $P = \frac{\pi^2 EI}{L^2}$

$$E = \frac{\sigma}{e} = \frac{P}{A} \div \frac{\Delta L}{L} = \frac{600000}{\frac{765.76}{6.4} \div \frac{5000}{6.4}} = \frac{783.53}{1.28 \times 10^3} = 61.21 \times 10^3 \text{ N/mm}^2$$

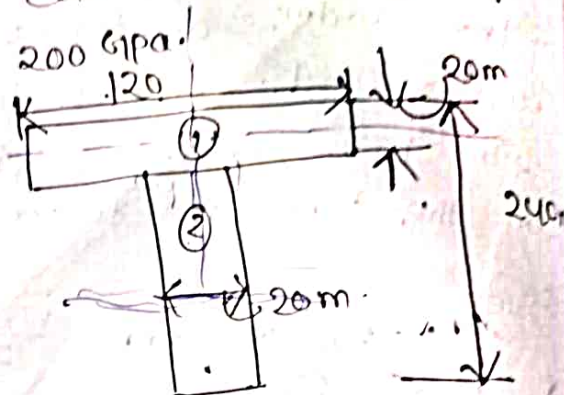
$$I = \frac{\pi}{64} (D^4 - d^4)$$

$$= \frac{\pi}{64} (40^4 - 25^4) = 106.48 \times 10^3$$

$$P = \frac{\pi^2 \times 61.21 \times 10^3 \times 106.48 \times 10^3}{2500} = 257.30 \times 10^3$$



Q3// A T-section $240\text{ mm} \times 120\text{ mm} \times 20\text{ mm}$ is used as 6 m long column with both ends fixed, what is the crippling load for the column? Take young's modulus for the joist as 200 GPa .



In rectangle-①

$$a_1 = 120 \times 20 = 2400\text{ mm}^2$$

$$x_1 = \frac{120}{2} = 60\text{ mm}$$

$$y_1 = 240 - 10 = 230\text{ mm}$$

In rectangle-②

$$a_2 = 220 \times 20 = 4400\text{ mm}^2$$

$$y_2 = \frac{220}{2} = 110\text{ mm}$$

$$x_2 = \frac{120}{2} = 60\text{ mm}$$

$$\bar{y} = \frac{2400 \times 230 + 4400 \times 110}{2400 + 4400}$$

$$= 152.35\text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{2400 \times 60 + 4400 \times 60}{2400 + 4400} = 60\text{ mm}$$

$$I_{xx} = I_{xx1} + I_{xx2}$$

$$I_{xx1} = I_{c1} + a_1 h_1^2$$

$$= \frac{bd^3}{12} \times a_1 \times (230 - 152.35)^2$$

$$= \frac{120 \times 20^3}{12} + (2400 \times 77.65^2)$$

$$= 14.54 \times 10^6\text{ mm}^4$$

$$I_{xx2} = \frac{20 \times 220^3}{12} + (4400 \times (152.35 - 11.0^2))$$

$$= 28.63 \times 10^6 \text{ mm}^4$$

$$I_{xx} = I_{xx1} + I_{xx2}$$

$$= 14.84 \times 10^6 + 28.63 \times 10^6$$

$$= 40.188 \times 10^6 \text{ mm}^4$$

$$I_{yy} = I_{yy1} + I_{yy2}$$

$$I_{yy1} = I_{yy11} + a_1 h_{11}^2$$

$$= \frac{db^3}{12} + a_1 h_{11}^2$$

$$= \frac{20 \times 120^3}{12} + 2400 \times (0)^2 = 2.88 \times 10^6 \text{ mm}^4$$

$$I_{yy12} = I_{yy12} + a_2 h_{12}^2$$

$$= \frac{220 \times 20^3}{12} + 4400 \times 10^2$$

$$= 146.66 \times 10^3 \text{ mm}^4$$

$$I_{yy} = I_{yy1} + I_{yy12}$$

$$= 2.88 \times 10^6 + 146.66 \times 10^3$$

$$= 3.02 \times 10^6 \text{ mm}^4$$

$$P = \frac{\pi^2 EI}{L^2}$$

$$L = L/2 = \frac{6000}{2} = 3000 \text{ mm}^2$$

$$P = \frac{\pi^2 \times 200 \times 10^3 \times 3.02 \times 10^6}{3000^2}$$

$$= 662.36 \times 10^3 \text{ N}$$

$$= 662.36 \text{ kN}$$

Shear force & bending moment

⇒ Shear force → The shear force at the cross-section of the beam is defined as the unbalanced vertical force to the right or left of the section.

(or)

→ The algebraic sum of the vertical forces to the right or left of the cross-section of a beam.

⇒ Beam!- A structural member which is set upon by a system of external load at right angle to its axis is known as beam.

⇒ Bending moment!- The bending moment at the cross-section of a beam is defined as the algebraic sum of the moments of the forces, to the right or left of the section.

$$\text{Moment} = \text{force} \times \perp \text{ distance}$$

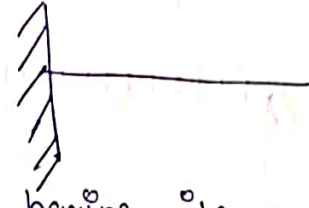
DATE 9.11.2022

⇒ Types of beam!- The types of beam are classified under different category such as :-

1. Cantilever beam.
2. Simple supported beam.
3. Fixed beam.
4. Continuous beam
5. ~~A beam which is fixed at~~ over hanging beam.

→ Simple supported beam!-

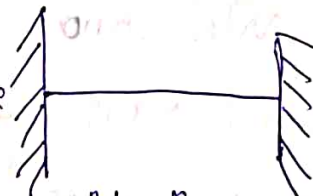
⇒ cantilever beam:- A beam which is fixed at one end & free at the other end. is called cantilever beam.



⇒ simple supported beam:- A beam having its both ends freely resting on walls or columns is called simple supported beam.

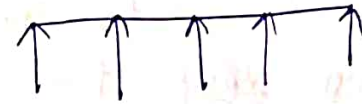


⇒ Fixed beam:- A beam with both ends rigidly fixed or built in is called fixed beam.



RCC beam in a frame structure

⇒ continuous beam:- A beam which is supported over more than two supports is called continuous beam.



⇒ Over hanging beam:- A beam which is simple supported at two points & projected at one or both ends beyond the support is called over hanging beam.

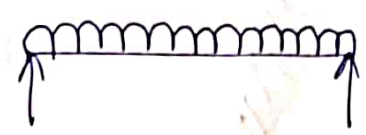


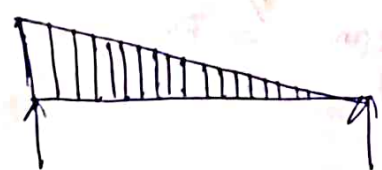
⇒ Types of load:-

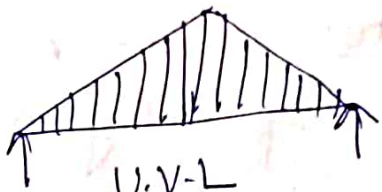
⇒ A beam may be subjected to either or combination of the following types of load. such as:-

1. point load or concentrated load.
2. uniformly distributed load. (UDL)
3. uniformly varying load (UVL)

1// Point load 

2// UDL 

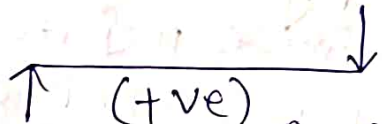
3// UVL 


U.V.L

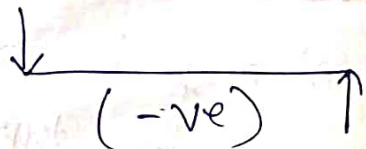
⇒ Sign convention for shear force & bending moment:-

⇒ S.F → We know that as the shear force is unbalanced vertical force therefore it tends to slide 1 person of the beam, upward & downward with respect to other.

⇒ We take the shear force at the section as positive when left hand person tends to slide upward, or right hand person tends to slide downward.


(+ve)

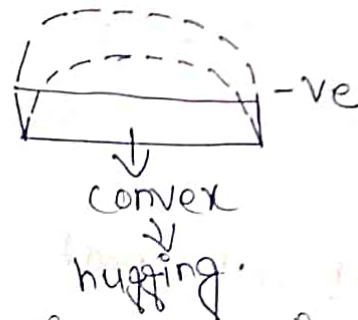
⇒ Similarly we take the S.F at the section is negative when the left hand person tends to slide downward or right hand person tends to slide upward.

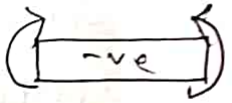
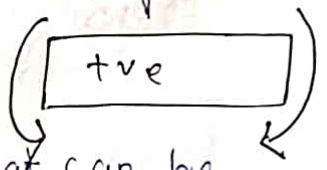

(-ve)

⇒ Bending moment:- We take the bending moment at a section as +ve if it tends to bend the beam at a point to a curvature having concavity at the top. Concave sagging.


+ve

- On the other hand we take the bending moment at the section as negative if it tends to bend.
- The beam at the top to a curvature having convexity at the top.



- We often call the positive bending moment is sagging moment & the negative bending moment is hugging moment.
- In another way of assigning the sign convention of the bending moment is by the direction in which it acts at a section, we take the bending moment at a section as +ve when it is acting in clock wise direction to the left or anticlock wise direction to the right. 
- On other hand we take the bending moment at a section as negative when it is acting in anticlock wise direction to the left or in clock wise direction to the right. 
- Statically Determinate structure:-

- A statically Determinate structure is one that can be analysed and the reaction & the forces determined from the eqn of equilibrium.
- The eqn's are $\sum H = 0$, $\sum V = 0$ & $\sum M = 0$
- If $r = 3n$ it is said to be statically determinate when r is total parts of the structural member & n is total nos of unknown reactive force or moment.

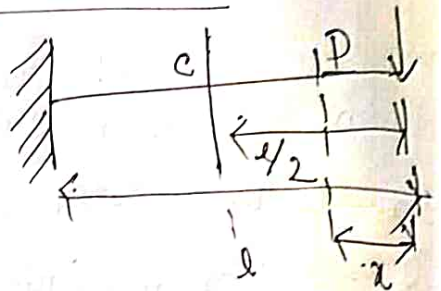
→ Statically Indeterminate structure:-

→ A member or a structure that cannot be analysed the eqn of equilibrium.

→ The eqns are $\sum H \neq 0$, $\sum V \neq 0$ & $\sum M \neq 0$

→ If n is $> 3n$ it should be statically underdeterminate.

→ Cantilever with point loads at its free end:-



→ Shear force calculation:-

Shear force at B = $+W$

Shear force at D = $+W$

Shear force at A = $+W$

→ Bending moment calculation:-

Bending moment at B = force $\times$ perpendicular distance = $+W \times 0 = 0$

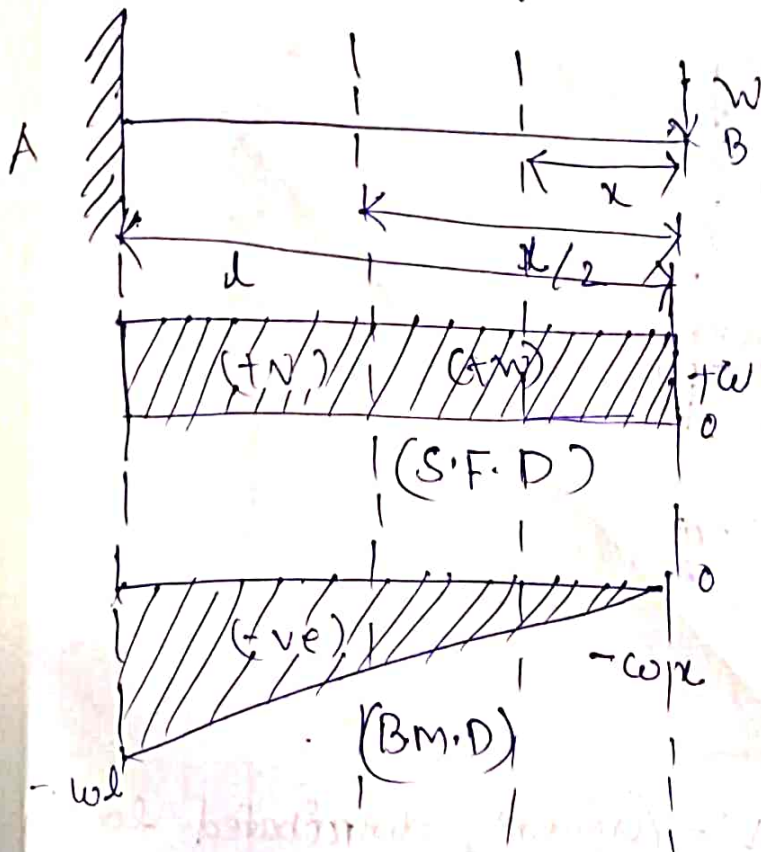
Bending moment at D = force $\times$ $\perp$ distance
= $-W \times x$

Bending moment at A = force $\times$ $\perp$ distance

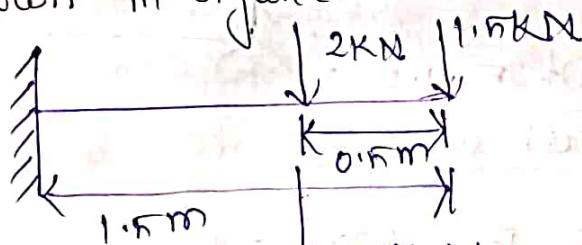
$$= -W \times l$$

$$= -wl$$

Shear force bending moment diagram:-



Q. Draw the shear force and bending moment diagram for a cantilever beam of span 1.5m carrying point loads as shown in figure.



A) Shear force at B = 1.5kN
 Shear force C = $1.5 + 2 = 3.5\text{kN}$
 Shear force A = 3.5kN

Bending moment at B = force $\times$ distance = 0

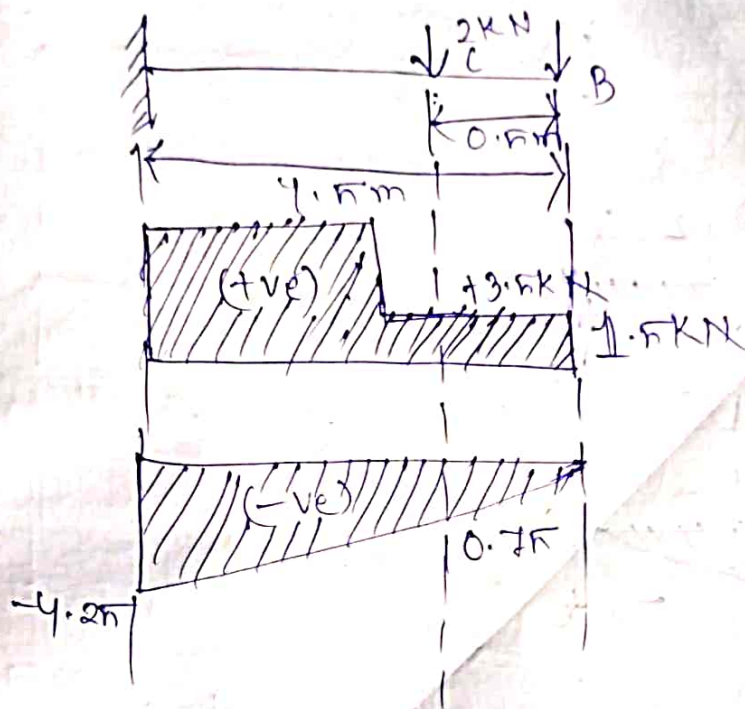
$$B.M_C = (1.5 \times 0.5) - (2 \times 1)$$

$$= 0.75\text{ kN}\cdot\text{m}$$

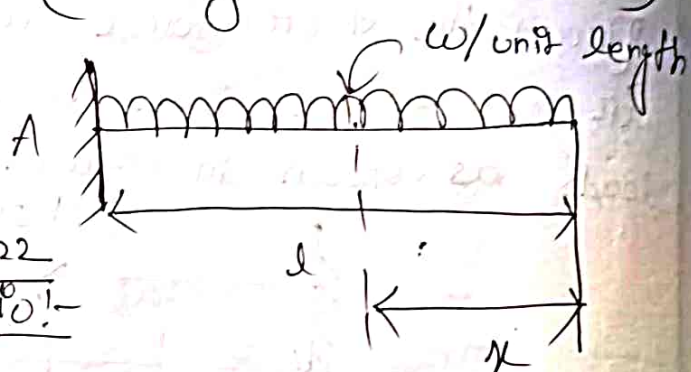
$$B.M_A = (1.5 \times 1.5) + (2 \times 1)$$

$$= -4.25$$

→ Shear force bending diagram



→ Cantilever with UDL:- (uniformly distributed load)



→ Shear force calculation:-

$$\text{Shear force at B} = w \times 0 = 0$$

$$\text{S.F.C at } x = w \times x = wx$$

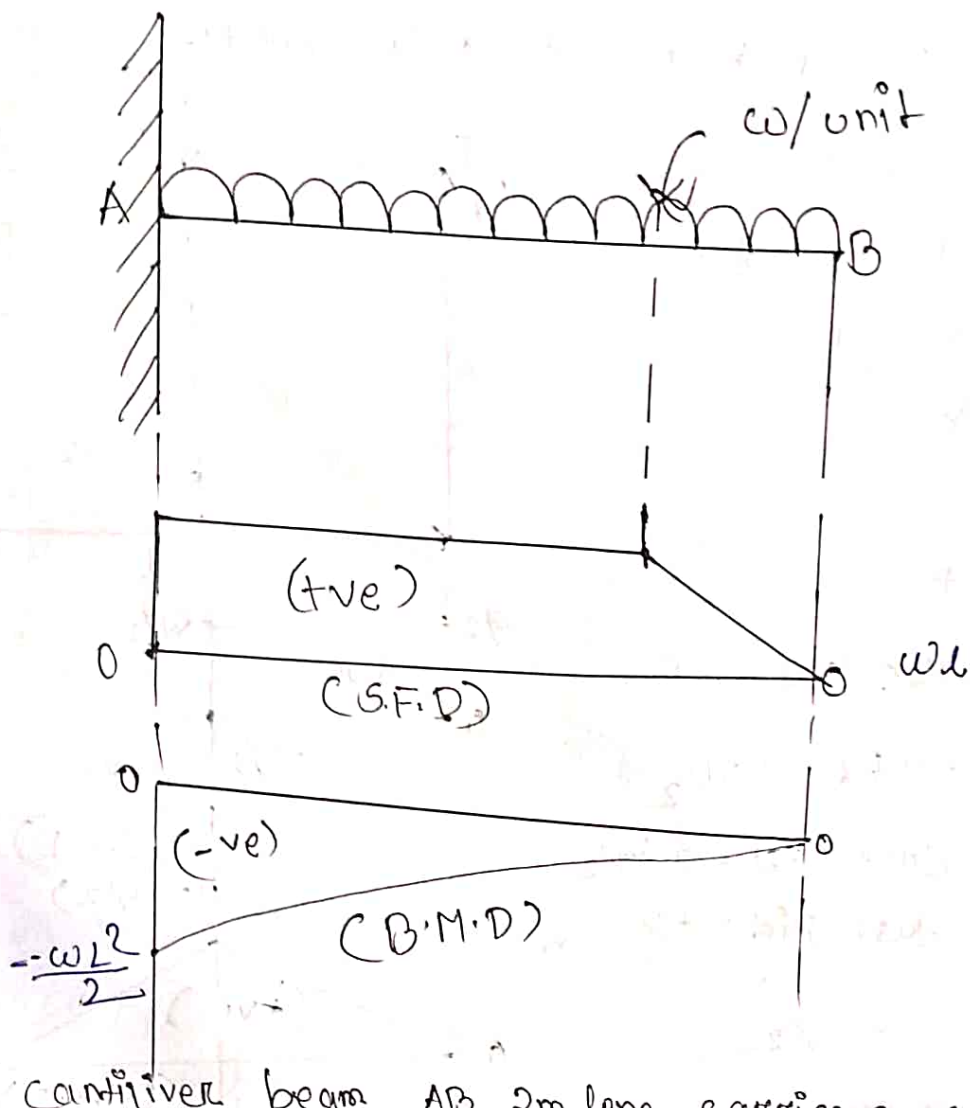
$$\text{S.F.A at A} = w \times l = wL$$

→ Bending moment calculation:-

$$\text{B.M at B} = 0$$

$$\text{B.M at C} = w \times x \times \frac{x}{2} = \frac{wx^2}{2}$$

$$\text{B.M at A} = w \times L \times \frac{L}{2} = \frac{wL^2}{2}$$



Q1// A cantilever beam AB 2m long carries a uniformly distributed load of 1.5 kN/m over a length of 1.6 m from the Free end. Draw the shear force & bending moment diagram for the beam.

Given

$$AB = 2 \text{ m}$$

$$\text{UDL} = 1.5 \text{ kN/m}$$

$$L = 1.6 \text{ m}$$

$$\text{S.F at B} = 0$$

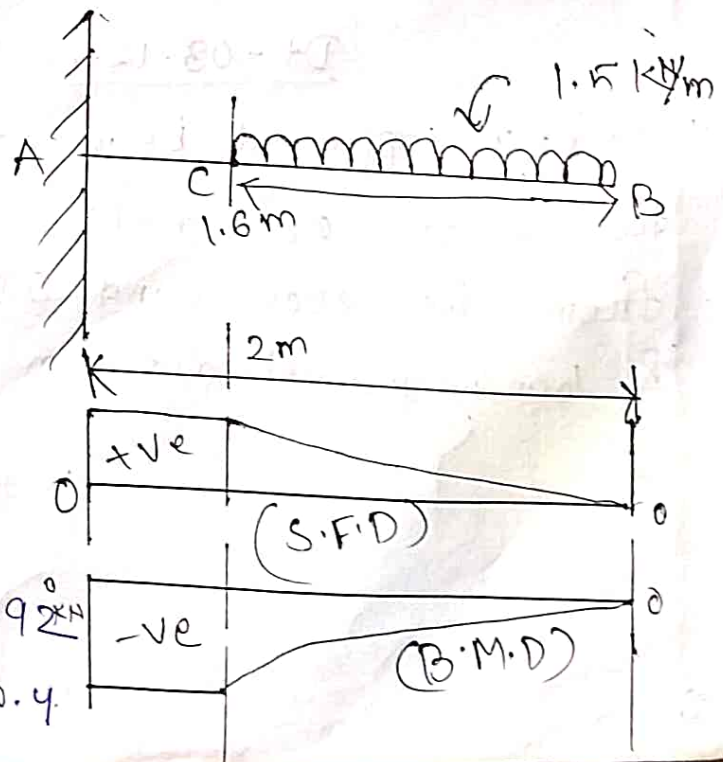
$$\text{S.F at C} = 1.5 \text{ kN} \times 1.6 = 2.4 \text{ kN}$$

$$\text{S.F at A} = 2.4 \text{ kN}$$

$$\text{B.M at B} = 0 = 1.5 \times 0$$

$$\text{B.M at C} = (1.5 \times 1.6) \times \frac{1.6}{2} = 1.92 \text{ kN}$$

$$\begin{aligned} \text{B.M at A} &= (1.5 \times 1.6) \times \frac{1.6}{2} + 0.4 \\ &= -2.88 \text{ kN m} \end{aligned}$$



Dt-26.11.2022

Simply supported beam with point load:-

→ Σ of upward force = Σ of downward force

$$R_A + R_B = W$$

$$\rightarrow R_A + R_A = W$$

$$\rightarrow 2R_A = W$$

$$\rightarrow R_A = \frac{W}{2} = R_B$$

Shear force calculation

$$\rightarrow \text{S.F at A} = +R_A = +W/2$$

$$\rightarrow \text{S.F at C just left} = +W/2$$

$$\rightarrow \text{S.F at C just Right} = +\frac{W}{2} - W$$

$$\rightarrow \text{S.F at B} = -W/2$$

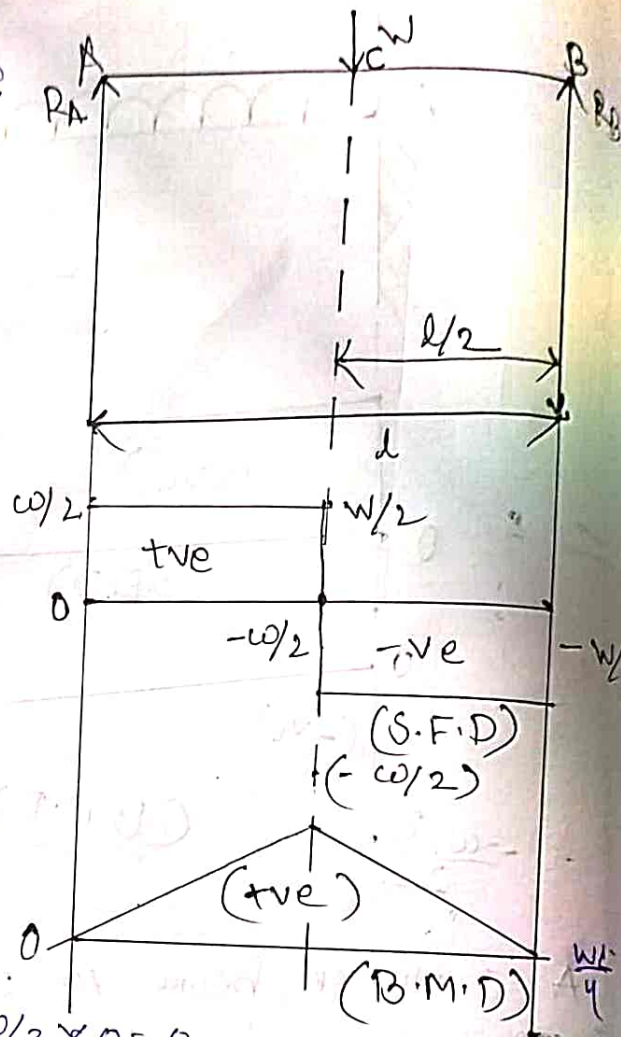
Bending moment at A = $W/2 \times 0 = 0$

$$\rightarrow \text{B.M at C} = \left(+\frac{W}{2} \times \frac{l}{2} \right) = \frac{Wl}{4}$$

$$\rightarrow \text{B.M at B} = +\left(\frac{W}{2} \times l \right) - \left(W \times \frac{l}{2} \right) = 0$$

Dt-03.12.2022

Q1// A simply supported beam of 3m span. Carried two load of 5kN each at 1m & 2m from the left hand support draw the shear force & bending moment diagram for the beam?



taking moment about A
& equating the sum

$$-R_B \times 3 + (5 \times 2) + (5 \times 1) = 0$$

$$\Rightarrow (10 + 5) = R_B \times 3$$

$$\Rightarrow R_B = \frac{15}{3} = 5 \text{ kN}$$

$$R_A + R_B = 5 + 5$$

$$\Rightarrow R_A = 10 - 5 = 5 \text{ kN}$$

★ Shear force calculation

$$\text{S.F at A} = +5 \text{ kN}$$

$$\text{S.F at C, just left} = +5 \text{ kN}$$

$$\text{S.F at C, just Right} = +5 \text{ kN} - 5 \text{ kN} = 0$$

$$\text{S.F at D, just left} = 0$$

$$\text{S.F at D, just Right} = 0 - 5 \text{ kN} = -5 \text{ kN}$$

$$\text{S.F at B} = -5 \text{ kN}$$

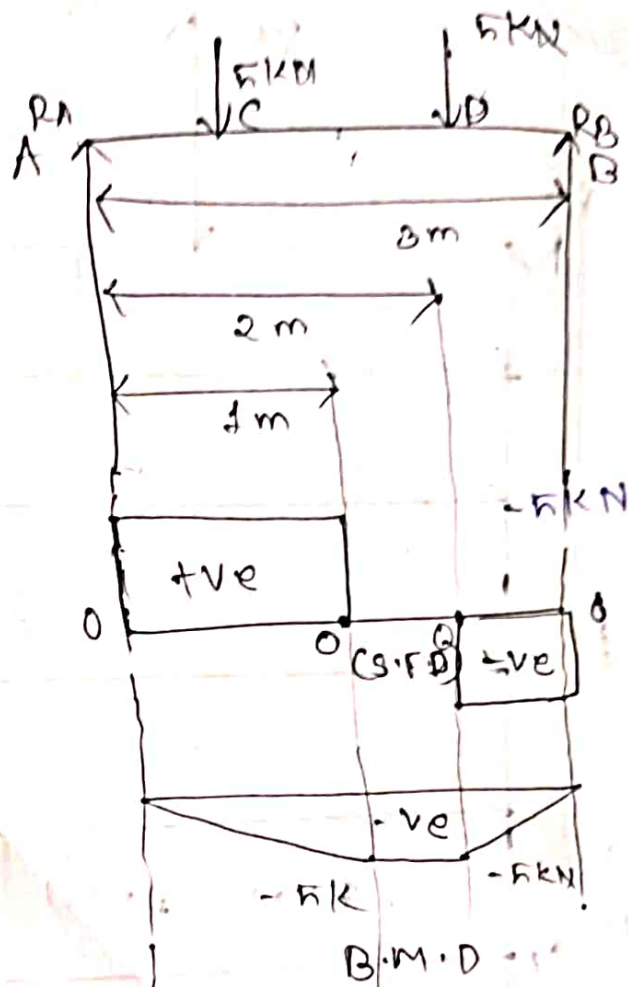
★ Bending moment calculation

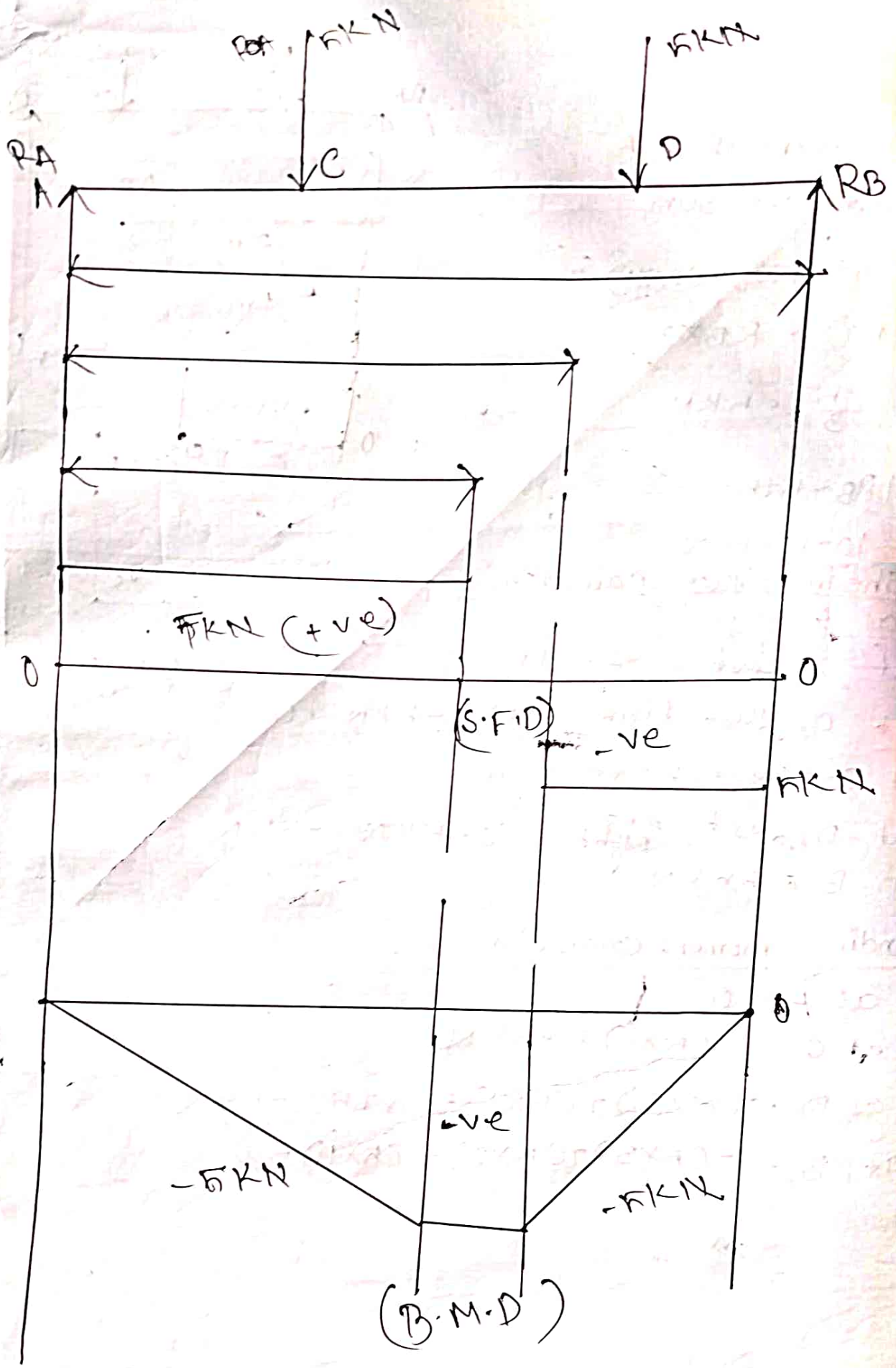
$$\text{B.M at A} = 0$$

$$\text{B.M at C} = -(5 \times 1) = -5 \text{ kNm}$$

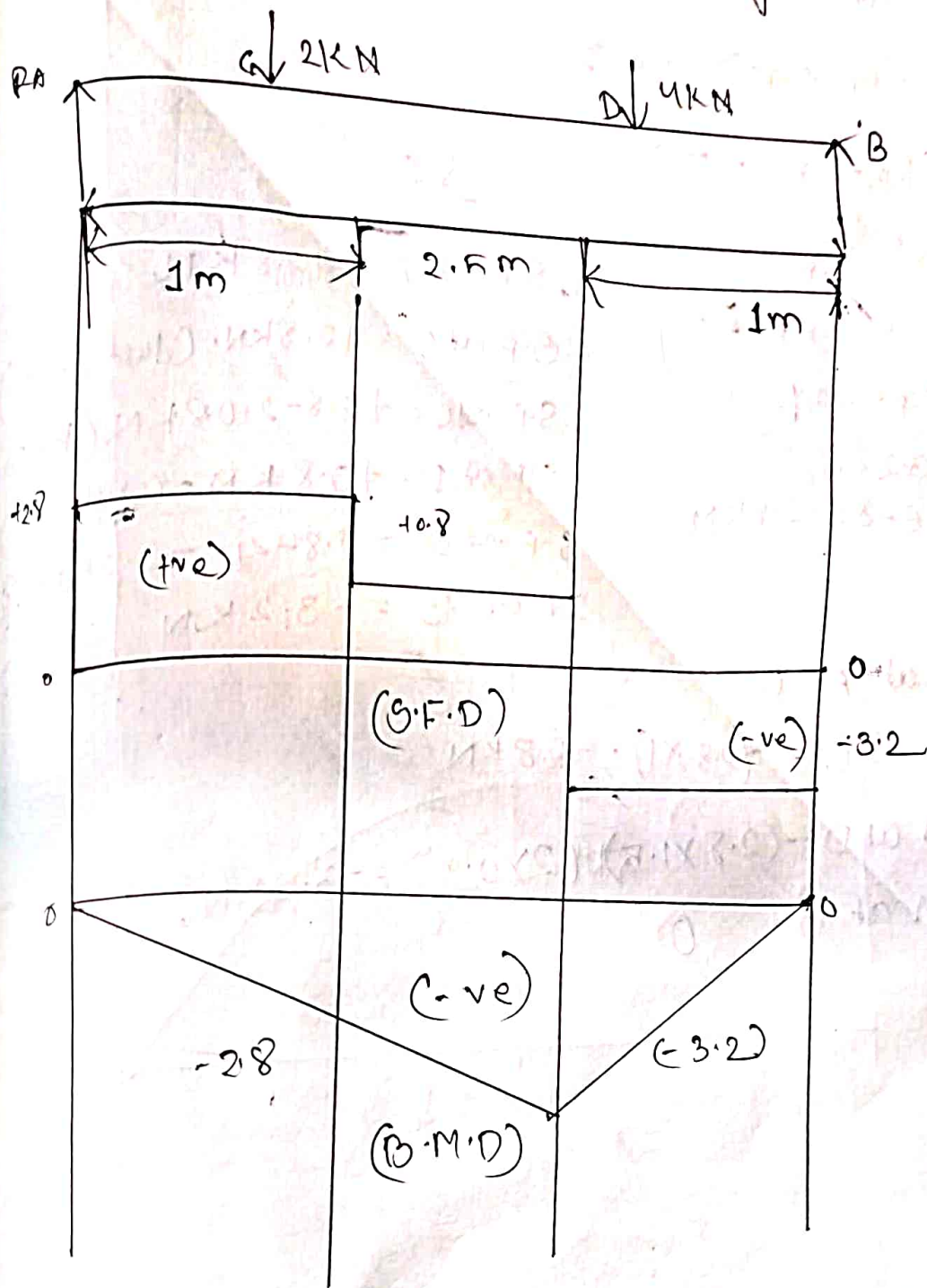
$$\text{B.M at D} = -(5 \times 2) + (5 \times 1) = -10 + 5 = -5 \text{ kNm}$$

$$\text{B.M at B} = -(5 \times 3) + (5 \times 2) + (5 \times 1) = 0$$





A simply supported beam AB of span 2.5 m is carrying 2 point loads as shown in figure.



Taking moment about A & equate the sum

$$-(R_B \times 2.5) + (4 \times 1.5) + (2 \times 1) = 0$$

$$\Rightarrow -(R_B \times 2.5) = -6 - 2$$

$$\Rightarrow -(R_B \times 2.5) = -8$$

$$\Rightarrow R_B = \frac{8}{2.5}$$

$$\Rightarrow R_B = 3.2 \text{ kN}$$

$$\Rightarrow R_A + R_B = 2 + 4$$

$$\Rightarrow R_A + 3.2 = 6$$

$$\Rightarrow R_A = 6 - 3.2 = 2.8 \text{ kN}$$

S.F

$$\text{S.F at A} = +2.8 \text{ kN}$$

$$\text{S.F at C} = +2.8 \text{ kN} \cdot (\text{Just left})$$

$$\text{S.F at C} = +2.8 - 2 = 0.8 \text{ kN} \cdot (\text{Just Right})$$

$$\text{S.F at D} = +0.8 \text{ kN} \cdot (\text{Just left})$$

$$\text{S.F at D} = +0.8 - 4 = -3.2 \text{ kN} \cdot (\text{Just Right})$$

$$\text{S.F at B} = -3.2 \text{ kN}$$

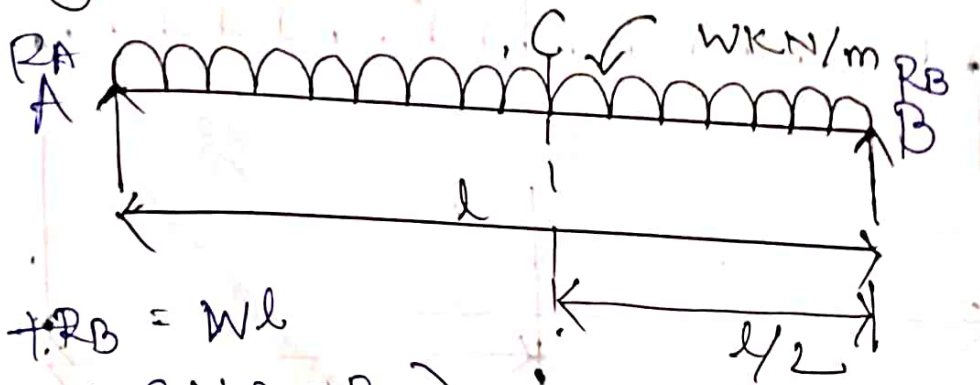
$$\text{B.M at A} = 0$$

$$\text{B.M at C} = -(2.8 \times 1) = -2.8 \text{ kN}$$

$$\text{B.M at D} = -(2.8 \times 1.5) + (2 \times 0.5) = -3.2 \text{ kN}$$

$$\text{B.M at B} = 0$$

Simply supported beam with UDL



$$R_A + R_B = w l$$

$$\Rightarrow 2R_A = w l \quad (\because R_A = R_B)$$

$$\Rightarrow R_A = \frac{w l}{2} = R_B$$

* Shear force calculation

$$\text{S.F at B} = - \frac{w l}{2}$$

$$\text{S.F at C} = - \frac{w l}{2} + \frac{w \times l}{2} = 0$$

$$\text{S.F at A} = - \frac{w l}{2} + \frac{w l}{2} = w l \left(-\frac{1}{2} + 1 \right) = \frac{1}{2} w l = \frac{w l}{2}$$

* Bending moment calculation

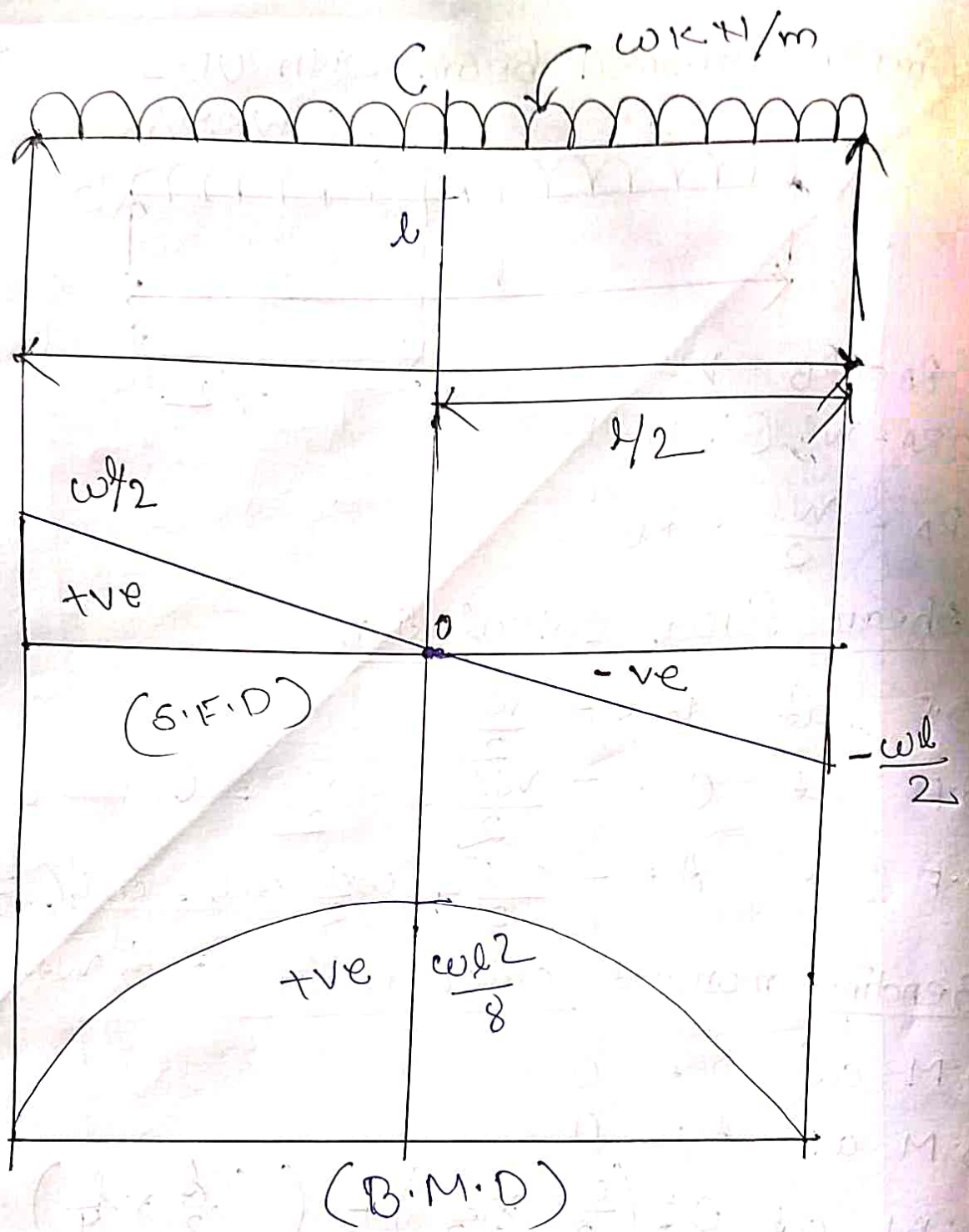
$$\text{B.M at B} = 0$$

$$\text{B.M at A} = 0$$

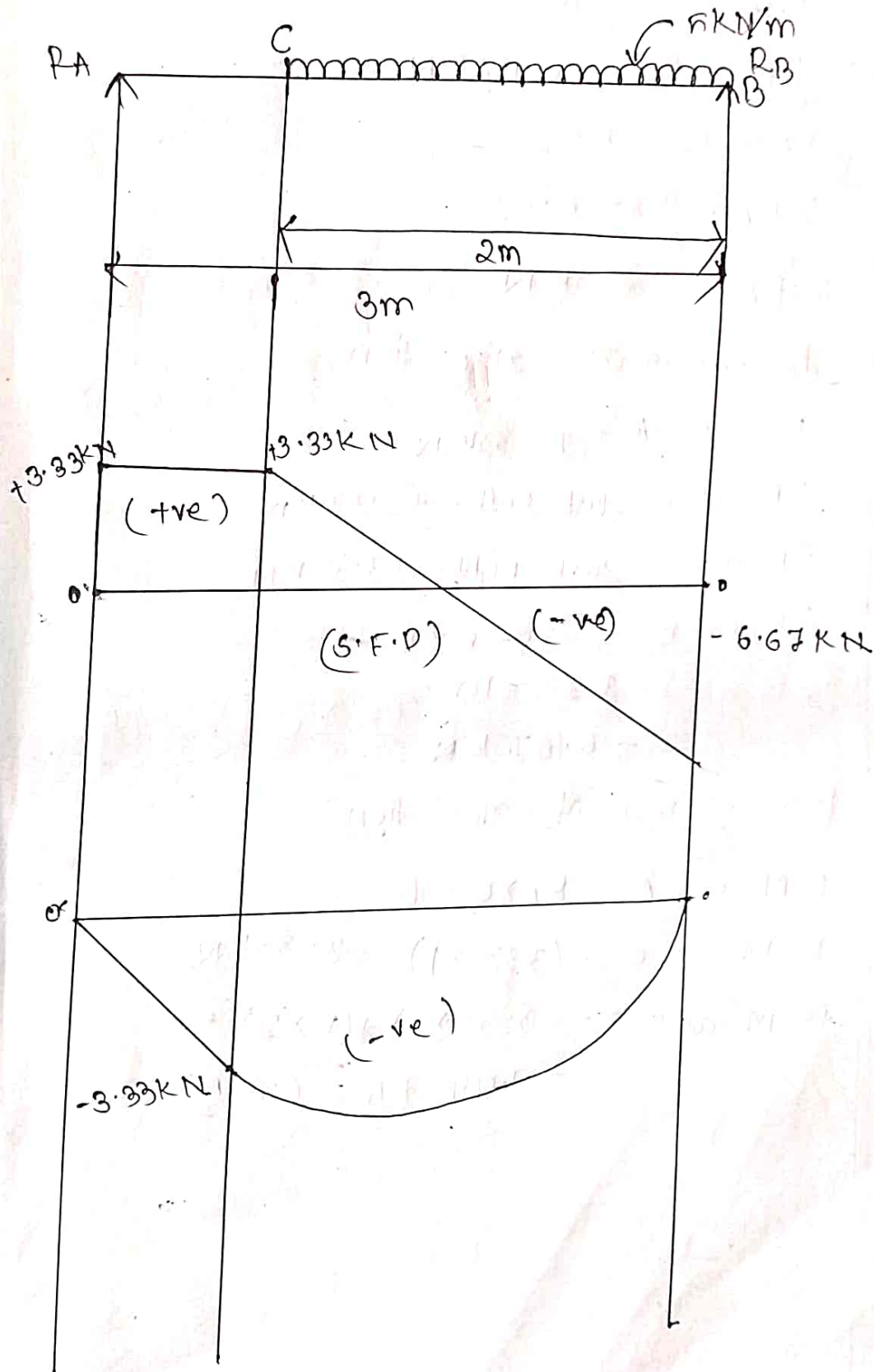
$$\text{B.M at C} = \left(\frac{w l}{2} \times \frac{l}{2} \right) - \left(\frac{w l}{2} \times \frac{l}{4} \right)$$

$$= - \frac{w l^2}{4} - \frac{w l^2}{8}$$

$$= \frac{-2w l^2 - w l^2}{8} = \frac{w l^2}{8}$$



Q// A simply supported beam carries 5 kN/m over a length of 2 m to the right hand support & span length of the beam 3 m - Draw the shear force & bending moment diagram.



Taking moment about A & equating the same

$$-(R_B \times 3) + (10 \times 2) \times \left(\frac{3}{2} + 1\right) = 0$$

$$= -(R_B \times 3) = -10 \times (1+1)$$

$$= R_B \times 3 = 10 \times 2$$

$$= R_B = \frac{20}{3} = 6.67 \text{ kN}$$

$$R_A + R_B = 10 \times 2$$

$$\Rightarrow R_A = 10 - 6.67$$

$$\Rightarrow R_A = 3.33 \text{ kN}$$

Shear force calculation

$$\text{S.F at A} = +3.33 \text{ kN}$$

$$\text{S.F at C Just left} = +3.33 \text{ kN}$$

$$\text{S.F at C Just Right} = +3.33 \text{ kN}$$

$$\begin{aligned} \text{S.F at B} &= +3.33 - 10 \times 2 \\ &= 3.33 - 10 \end{aligned}$$

$$= -6.67 \text{ kN}$$

Bending moment calculation

$$\text{B.M at A} = R_A \times 0 = 0$$

$$\text{B.M at C} = -(3.33 \times 1) = -3.33 \text{ kN}$$

$$\begin{aligned} \text{B.M at B} &= -(3.33 \times 3) + (10 \times 2) \\ &= -9.99 + 20 = 10.01 \text{ kN} \end{aligned}$$

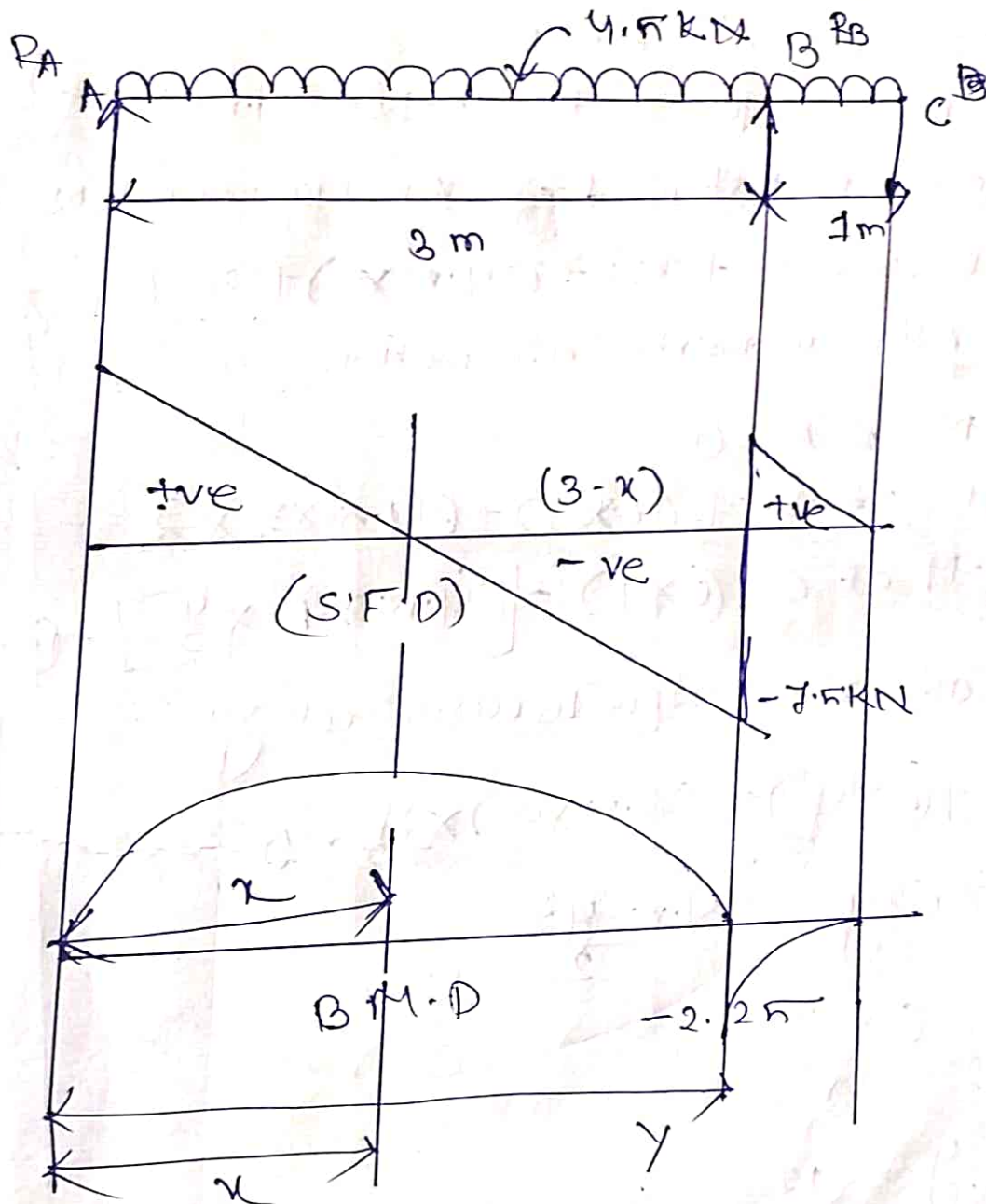
Dt-07.12.2022

⇒ point of contraflexure:-

→ where the bending moment changes sign (-ve to +ve or +ve to -ve) is known as point of contraflexure

→ The point of contraflexure is always arise on over hanging beam.

Q₂// When a over hanging beam ABC as shown in figure draw the shear force & bending moment diagram & find also the point of contraflexure if any.



Taking moment about A & equating, the same

$$+(R_B \times 3) - (4.5 \times 4) \times 2 = 0$$

$$\Rightarrow R_B = \frac{18 \times 2}{3} = \frac{36}{3} = 12$$

$$R_A + R_B = (4.5 \times 4)$$

$$\Rightarrow R_A = 18 - 12 = 6 \text{ kN}$$

Shear Force Calculation

$$\text{S.F at A} = +6 \text{ kN}$$

$$\text{S.F at B just left} = +6 \text{ kN} - (4.5 \times 3) = -7.5 \text{ kN}$$

$$\text{S.F at B just right} = -7.5 + 12 = 4.5 \text{ kN}$$

$$\text{S.F at C} = +6 \text{ kN} - (4.5 \times 4) + 12 = 0$$

Bending moment Calculation

$$\text{B.M at A} = 0$$

$$\text{B.M at B} = + (6 \times 3) - (4.5 \times 3) \times \frac{3}{2} = -2.25$$

$$\text{B.M at C} = (6 \times 4) - \left[(4.5 \times 4) \times \frac{4}{2} \right] + (12 \times 1) = 0$$

Point of contraflexure at y

$$+(6 \times y) - (4.5 \times y) \times \frac{y}{2} = 0$$

$$\Rightarrow 6 \times y = 4.5 \times \frac{y^2}{2}$$

$$\Rightarrow \frac{6 \times 2}{4.5} = y$$

$$\Rightarrow y = \frac{12}{4.5} = 2.66 \text{ m}$$

Point of contraflexure of x

→ Finding the max bending moment will occur x & find the distance between A & M.

$$\frac{x}{6} \times \frac{3-x}{7.5}$$

$$\Rightarrow 7.5x = (3-x)6$$

$$\Rightarrow 7.5x = 18 - 6x$$

$$\Rightarrow 7.5x + 6x = 18$$

$$\Rightarrow 13.5x = 18 \Rightarrow x = \frac{18}{13.5} = 1.34 \text{ m}$$

$$\begin{aligned} \text{max Bending moment} &= (6 \times 1.34) - (4.5 \times 1.34) \times \frac{1.34}{2} \\ &= 3.999 \approx 4 \text{ kNm} \end{aligned}$$

Q// Simply supported beam with point load & UDL

A simply supported beam of 4m span is carrying load as shown in figure draw the shear force & bending moment diagrams for the beam?

→ Taking moment about A & equating the same

$$+ (R_B \times 4) - [(2 \times 1) \times (\frac{1}{2} \times 1.5)] - (4 \text{ kN} \times 1.5) = 0$$

$$\Rightarrow (R_B \times 4) = 6 + 4$$

$$\Rightarrow R_B = \frac{10}{4} = 2.5 \text{ kN}$$

Σ of ~~up~~ up ward forces = Σ of down ward forces

$$R_A + R_B = 4 + (2 \times 1)$$

$$\Rightarrow R_A = 6 - 3.5 \text{ kN}$$

S.F at B = -2.5 kN

S.F at D just right = -2.5 kN

S.F at D just left = -2.5

S.F at C just right = $-2.5 + (2 \times 1) = 0.5 \text{ kN}$

S.F at C just left = $+0.5 + 4 = 3.5 \text{ kN}$

S.F at A = 3.5 kN

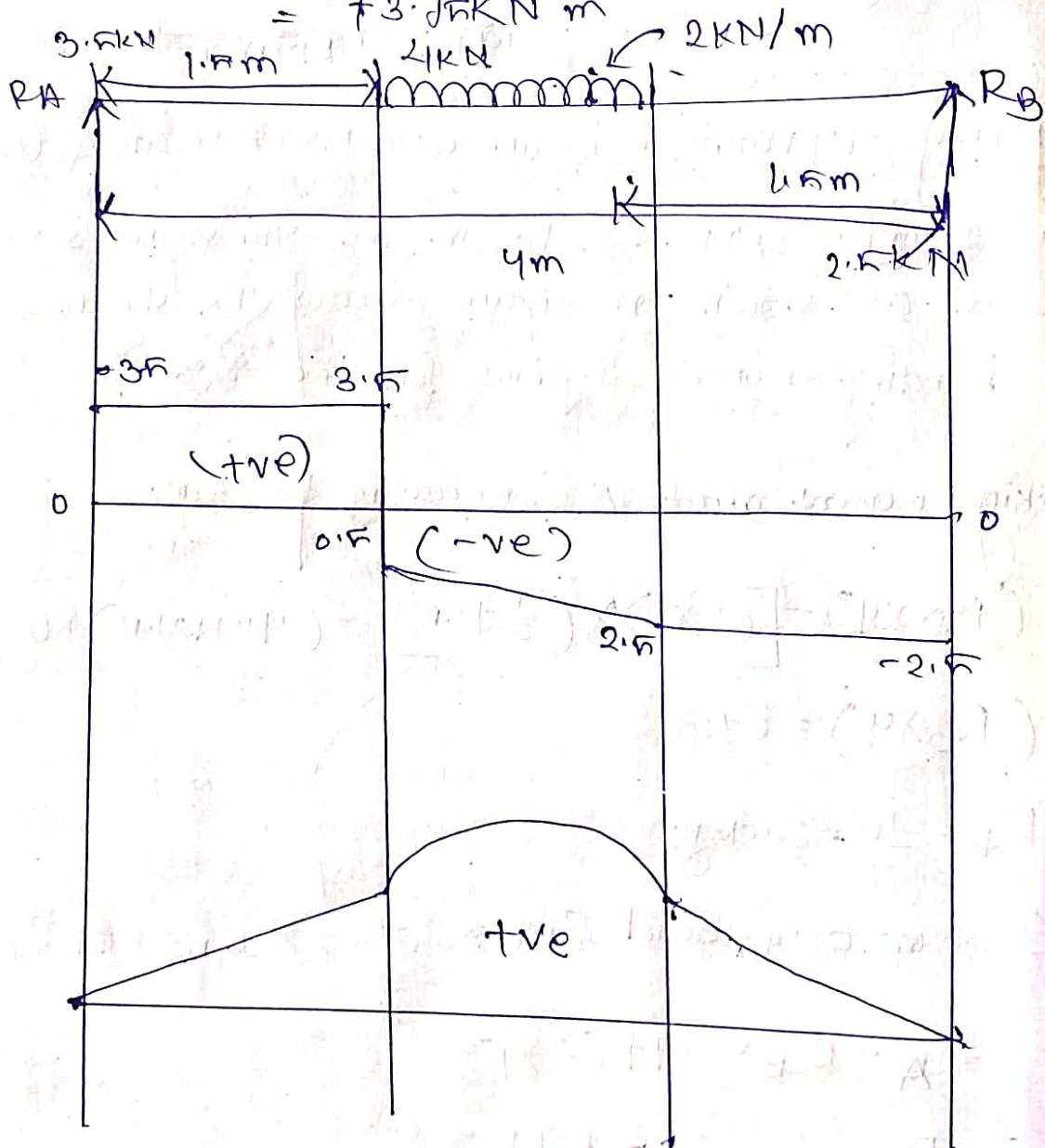
B.M calculation

B.M at A = B = 0

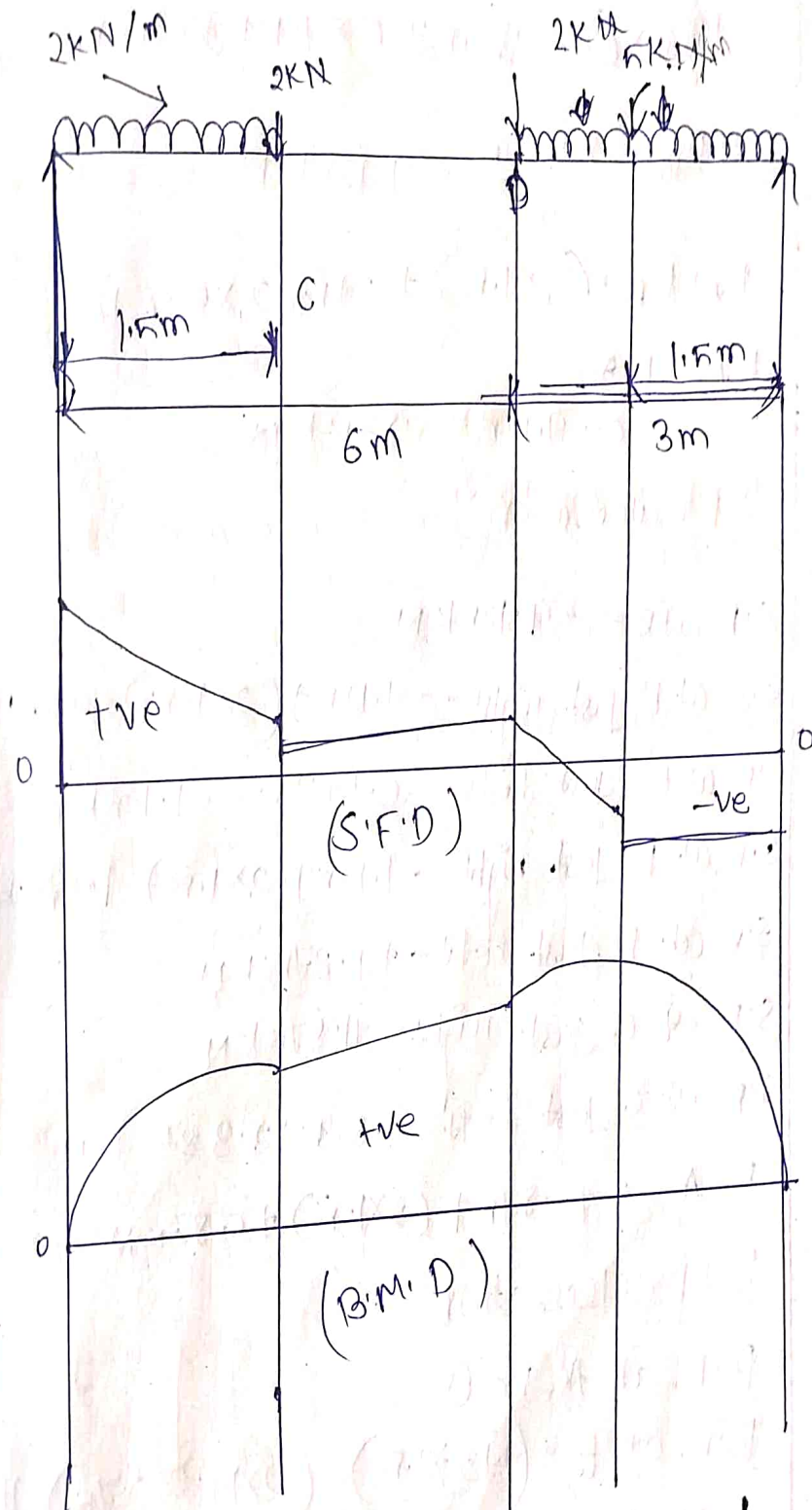
B.M at C = $+ (3.5 \times 1.5) = (5.25 \text{ kN})$

B.M at D = $+ (3.5 \times 2.5) - 4 \times 1 - \left[(2 \times 1) \times \frac{1}{2} \right]$

= +3.75 kN m



Q2/ A simply supported beam AB 6m long, is loaded as shown in figure. Draw the shear force & bending moment diagram for the beam.



$$= + (R_B \times 6) - (5 \times 4.5) - \left[(2 \times 3) \times \frac{3}{2} + 3 \right] - (2 \times 1.5) - (2 \times 1.5) \frac{1.5}{2} = 0$$

$$= + (R_B \times 6) - 22.5 + 6 \times 4.5 + 3 + 2.25$$

$$= + R_B = \frac{54.75}{6} = 9.125 \text{ kN}$$

$$= R_A + R_B = (2 \times 1.5) + 2 + (2 \times 3) = 16$$

$$R_A = 16$$

$$16 - 9.125 = 6.875 \text{ kN}$$

S.F Calculation

$$\text{S.F at B} = -9.125 \text{ kN}$$

$$\text{S.F at E just right} = -9.125 + (2 \times 1.5) = -6.125$$

$$\text{S.F at E just left} = -6.125 + 5 = -1.125 \text{ kN}$$

$$\text{S.F at D just right} = -1.125 + (2 \times 1.5) = 1.875 \text{ kN}$$

$$\text{S.F at D just left} = +1.875 \text{ kN}$$

$$\text{S.F at C just right} = +1.875 \text{ kN}$$

$$\text{S.F at C just left} = 1.875 + 2.387 = 4.262 \text{ kN}$$

$$\text{S.F at A} = +3.875 + (2 \times 1.5) + 6.875 \text{ kN}$$

B.M Calculation

$$\text{B.M at A \& B} = 0$$

$$\text{B.M at E} = (9.13 \times 1.5) - (2 \times 1.5 \times 0.75) = 11.44 \text{ kNm}$$

$$\begin{aligned} \text{B.M at D} &= (9.13 \times 3) - (5 \times 1.5) - (2 \times 3 \times 1.5) \\ &= 10.89 \text{ kNm} \end{aligned}$$

$$\begin{aligned} \text{B.M at C} &= (9.13 \times 4.5) - (5 \times 3) - [(2 \times 3) \times 1.5] \\ &= 8.085 \text{ kNm} \end{aligned}$$

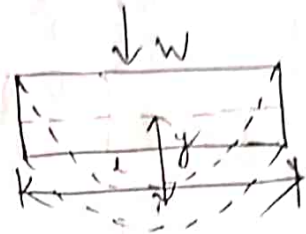
SLOPE AND DEFLECTION Dt = 16.12.2022

* DEFLECTION! - It is a distance of vertical distance of the beam measure after and before loading is known as deflection.

→ It is denoted by 'y'

→ Deflection of a support is always '0' & the loading part is maximum.

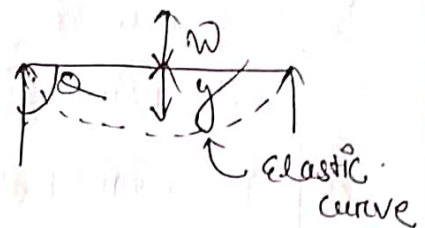
→ Its unit is mm.



* Slope! - It is the angle in radian measure between the tangent to elastic curve & original axis of the beam.

→ It is denoted by θ or $\frac{dy}{dx}$

→ Its unit is radian.



* Boundary Condition! -

→ Simply supported beam.

→ at point A & B deflection is = 0

$$L=0, y=0$$

$$L=L, y=0$$

$$\boxed{L=L, y=0}$$

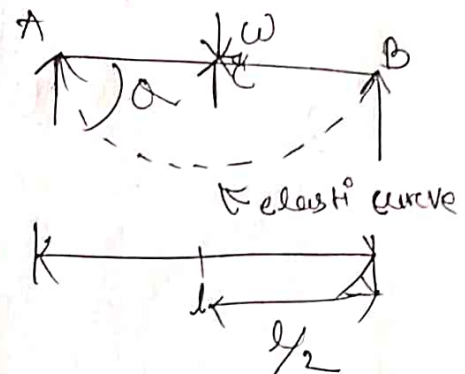
→ at point C Deflection is maximum

$$C = \frac{L}{2} \because y = y_{\max}$$

→ at point C the slope is (minimum) or (zero)

$$\text{the slope is } = 0$$

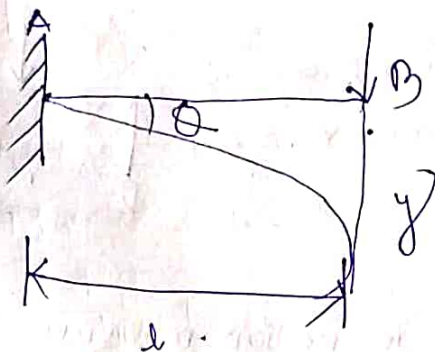
$$\rightarrow L = \frac{L}{2}, \theta = 0$$



→ at point a & b the slope is maximum

$$L=0 \text{ \& } L=L, \theta = \theta_{\text{max}}$$

* Cantilever beam:-



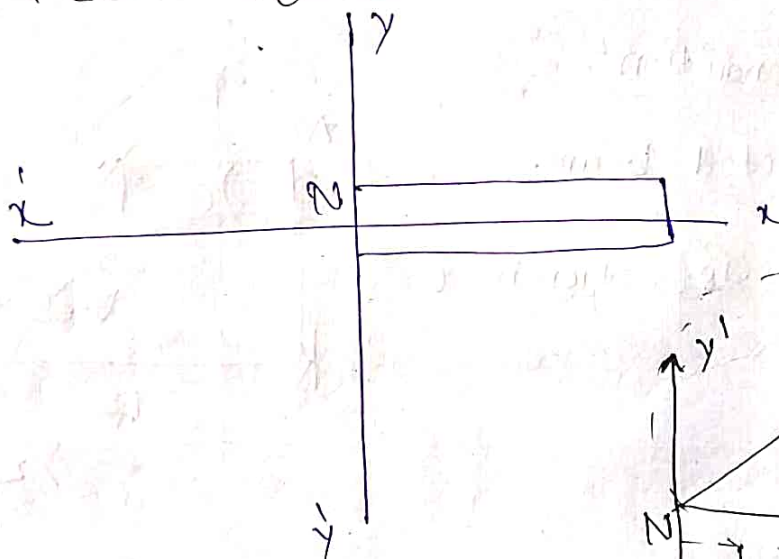
at point A the deflection = 0

at point A slope is always = 0

at point B Deflection is maximum.

at point B slope also maximum.

* Relation between slope & Deflection & elastic curve



From the elementary calculation

$$\frac{1}{R} = \frac{d^2y}{dx^2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}^{3/2} \quad \text{--- (1)}$$

where $\frac{dy}{dx}$ = slope --- (1)

and y is deflection.

slope & small deflection $\frac{dy}{dx}$ similar to ≈ 0

$$\boxed{\frac{1}{R} = \frac{d^2y}{dx^2}} \quad \text{--- (2)}$$

N.B :- here slope is maximum but deflection is very small that is 'y' is $y \approx 0$

we know the bending equation = $\frac{M}{I} = \frac{E}{R} \cdot \frac{1}{y}$

$$\Rightarrow \frac{M}{I} = \frac{E}{R}$$

$$= \frac{1}{R} = \frac{M}{EI} \quad \text{--- (iii)}$$

From the equation (2) & (3)

$$\frac{1}{R} = \frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\Rightarrow \boxed{M = \frac{d^2y}{dx^2} \times EI}$$

where M is bending moment
 EI ... flexural rigidity

F = shear force

Take differentiate both side with respect to x

$$\Rightarrow \text{we get } \frac{d}{dx} M = \frac{d}{dx} \left(\frac{d^2y}{dx^2} \right) EI$$

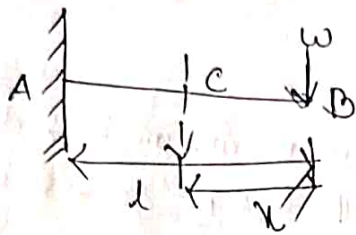
$$\boxed{F = EI \frac{d^3y}{dx^3}}$$

where F = shear force of the beam,

again differentiate both side with respect to x

$$\text{we get } \frac{d}{dx} F = \frac{d}{dx} \left(\frac{d^3y}{dx^3} \right) EI$$

where, w = load intensity per unit length of the beam
 $Dt = 17.12.2022$



B.M at $x = -wx \cdot x$

$(M)x = -wx$

$\frac{d^2y}{dx^2} EI = -wx$

Integration both side w.r.t x

We get

$\Rightarrow \frac{d^2y}{dx^2} EI = -wx$

$\Rightarrow \int \frac{d^2y}{dx^2} EI = \int -wx$

$\Rightarrow \frac{dy}{dx} EI = -w \frac{x^2}{2} + C_1$ — (1)

Integrate both side w.r.t x we get

$\Rightarrow \int \frac{dy}{dx} EI = \int -\frac{wx^2}{2} + C_1$

$\Rightarrow EI y = -\frac{w}{2} \times \frac{x^3}{3} + C_1 x + C_2$ — (2)

So this method is double integration method.

Boundary Condition:-

at $x=l$, $y=0$ & $\frac{dy}{dx} = 0$

Put the value in equation (1) & (2) in equation (1)

$\Rightarrow \frac{dy}{dx} EI = -w \frac{x^2}{2} + C_1 = 0$

$\Rightarrow 0 \times EI = -w \frac{l^2}{2} + C_1$

$\Rightarrow 0 = -w \frac{l^2}{2} + C_1 \Rightarrow \frac{wl^2}{2} = C_1$

Put the value in eqn ②

$$\Rightarrow E I y = -\frac{w}{2} \times \frac{x^3}{3} + C_1 x + C_2$$

$$\Rightarrow E I \times 0 = -\frac{w}{2} \times \frac{L^3}{3} + C_1 L + C_2$$

$$\Rightarrow 0 = -\frac{w}{2} \times \frac{L^3}{3} + \frac{w L^2}{2} \times L + C_2$$

$$\Rightarrow -\frac{w L^3}{6} + \frac{w L^3}{2} + C_2 = 0$$

$$\Rightarrow -\frac{w L^3 + 3 w L^3}{6} + C_2 = 0$$

$$\Rightarrow \frac{2 w L^3}{6} + C_2$$

$$\Rightarrow C_2 = -\frac{2 w L^3}{6}$$

$$\Rightarrow C_2 = -\frac{w L^3}{3}$$

Put the value of C_1 & C_2 in eqn ① & ② In eqn ①

$$\frac{dy}{dx} E I = -\frac{w x^2}{2} + C_1$$

$$\Rightarrow \frac{dy}{dx} E I = -\frac{w x^2}{2} + \frac{w L^2}{2} \text{ --- (3)}$$

In eqn ②

$$\Rightarrow E I y = -\frac{w}{2} \times \frac{x^3}{3} + C_1 x + C_2$$

$$\Rightarrow E I y = -\frac{w x^3}{6} + \frac{w L^2}{2} x - \frac{w L^3}{3} \text{ --- (4)}$$

It is the eqn of slope & deflection eqn but we calculate maximum slope and deflection so we take the boundary condition.

at $x=0$ & y & $\frac{dy}{dx}$ is max^m.

Put the value in eqn ② & eqn ④ In eqn ③

$$\frac{dy}{dx} E I = -\frac{w x^2}{2} + \frac{w L^2}{2}$$

$$\Rightarrow \frac{dy}{dx} E I = 0 + \frac{w L^2}{2}$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{\text{max}} = \frac{WL^2}{2EI} \quad \text{minimum slope}$$

In eqn (a)

$$EIy = -\frac{Wx^3}{6} + \frac{WL^2}{2}x - \frac{WL^3}{3}$$

$$\Rightarrow EIy = 0 + 0 - \frac{WL^3}{3}$$

$$\Rightarrow (y_{\text{max}}) = -\frac{WL^3}{3EI} \quad \text{Maximum deflection}$$

DT-3/1/23

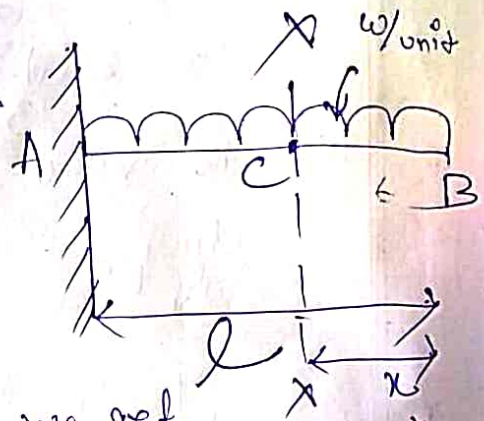


Cantilever beam with U.D.L

Bending moment at $x = -wx \times \frac{x}{2}$

$$M_{\text{max}} = -wx \frac{x^2}{2}$$

$$\Rightarrow \frac{d^2y}{dx^2} EI = -\frac{wx^2}{2}$$



Integrating both side w.r.t x we get

$$\int \frac{d^2y}{dx^2} EI = \int -\frac{wx^2}{2}$$

$$\Rightarrow EI \cdot \frac{dy}{dx} = -\frac{w}{2} \times \frac{x^3}{3} + C_1$$

$$\Rightarrow \boxed{EI \cdot \frac{dy}{dx} = -\frac{wx^3}{6} + C_1} \quad \text{--- (1)}$$

$$\Rightarrow EI \cdot \frac{dy}{dx} = -wx$$

Again integrate both side w.r.t x we get

$$\int EI \cdot \frac{dy}{dx} = \int -wx$$

$$\Rightarrow EIy = -\frac{w}{6} \times \frac{x^4}{4} + C_1x + C_2$$

$$\Rightarrow \boxed{EIy = -\frac{wx^4}{24} + C_1x + C_2} \quad \text{--- (2)}$$

Boundary Condition :-

When

$$x=L, y=0 \quad \frac{dy}{dx} = 0$$

Put the value in eqn ① & ②

In eqn ①

$$\text{EI} \cdot \frac{dy}{dx} = -\frac{wx^3}{6} + C_1$$

$$\Rightarrow 0 = -\frac{WL^3}{6} + C_1$$

$$\Rightarrow \boxed{C_1 = \frac{WL^3}{6}}$$

Put the C_1 value in eqn ②

$$\text{EI} y = -\frac{wx^4}{24} + C_1 x + C_2$$

$$\Rightarrow 0 = -\frac{WL^4}{24} + \frac{WL^3}{6} \times L + C_2$$

$$\Rightarrow C_2 = -\frac{3WL^4}{24} = \left(\frac{WL^4}{24} - \frac{WL^4}{6} \right)$$

$$\Rightarrow \boxed{C_2 = -\frac{WL^4}{8}}$$

Put the value of C_1 & C_2 in eqn ① & ② in eqn ①

$$\text{EI} \cdot \frac{dy}{dx} = -\frac{wx^3}{6} + C_1$$

$$\Rightarrow \boxed{\text{EI} \frac{dy}{dx} = -\frac{wx^3}{6} + \frac{WL^3}{6}} \quad \text{--- (3)}$$

In eqn ②

$$\text{EI} y = -\frac{wx^4}{24} + C_1 x + C_2$$

$$\boxed{\text{EI} y = -\frac{wx^4}{24} + \frac{WL^3}{6} x - \frac{WL^4}{8}} \quad \text{--- (4)}$$

Put the boundary condition in eqn ③ & ④ and get the maximum slope and deflection

at $x=0$, y & $\frac{dy}{dx}$ is maximum

In eqⁿ ③

$$EI \frac{dy}{dx} = -\frac{wx^3}{6} + \frac{wl^2}{6}$$

$$\left(\frac{dy}{dx} \right)_{\max} = \frac{wl^2}{6EI} \quad \text{maximum slope}$$

In eqⁿ ④

$$EIy = -\frac{wx^4}{24} + \frac{wl^2}{6} \times x - \frac{wl^3}{8}$$

$$\Rightarrow EIy = 0 \text{ to } -\frac{wl^3}{8}$$

$$\Rightarrow (y_{\max}) = -\frac{wl^3}{8EI} \quad \text{minimum deflection.}$$

Dt-4/1/23

Simply supported beam with point load:-

Find Reactions:-

We know that taking moment about A

$$R_B \times l - (W \times l/2) = 0$$

$$\Rightarrow R_B \times l = \frac{wl}{2}$$

$$\Rightarrow R_B = \frac{wl}{2} \times \frac{1}{l} = \frac{w}{2}$$

$$\Rightarrow R_B = \frac{W}{2}$$

$R_A + R_B = \text{Total load}$

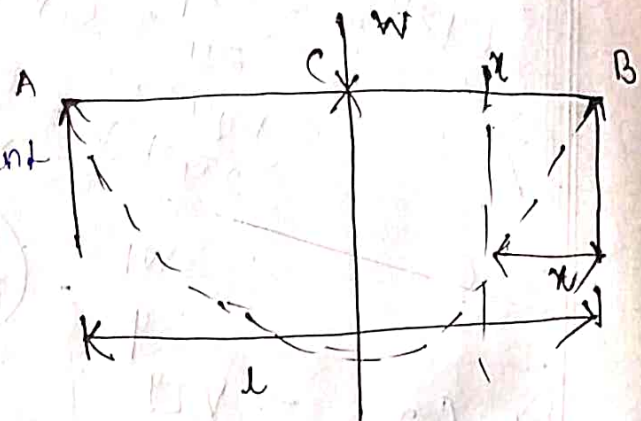
$$R_A + R_B = W$$

$$\Rightarrow R_A + \frac{W}{2} = W$$

$$\Rightarrow R_A = W - \frac{W}{2}$$

$$\Rightarrow R_A = \frac{2W - W}{2}$$

$$\Rightarrow R_A = \frac{W}{2} = R_B$$



Taking a section ($x-x$) in right side at a distance (x) from point B

Taking moment of $x-x$ section.

Bending moment at x

$$M_{\max} = P_B \times x$$

$$M_{\max} = \frac{w}{2} \times x$$

$$\Rightarrow \frac{d^2y}{dx^2} \cdot EI = \frac{w}{2} \times x$$

Integrating the above eqn we get

$$\int \frac{d^2y}{dx^2} EI = \int \frac{w}{2} \times x$$

$$\Rightarrow \frac{dy}{dx} = \frac{w}{2} \times \frac{x^2}{2} + C_1$$

$$\boxed{\Rightarrow EI = \frac{dy}{dx} = \frac{wx^2}{4} + C_1} \quad \text{--- (1)}$$

Again we have to find the value of constant by using boundary condition.

When $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$

In eqn (1)

$$EI \cdot \frac{dy}{dx} = \frac{wx^2}{4} + C_1$$

$$\Rightarrow 0 = \frac{w}{4} \times \frac{l^2}{4} + C_1$$

$$\boxed{\Rightarrow C_1 = -\frac{wl^2}{16}}$$

The value of C_1 put in eqn (1) $\Rightarrow EI \frac{dy}{dx} = \frac{wx^2}{4} + C_1$

$$\boxed{EI \cdot \frac{dy}{dx} = \frac{wx^2}{4} - \frac{wl^2}{16}} \quad \text{--- (2)}$$

This is the eqⁿ of slope but we calculate the maximum slope will be at point A & B. Put boundary condition,

if $x=0$, $\frac{dy}{dx}$ = maximum

The value put in eqⁿ (2)

$$EI \frac{dy}{dx} = \frac{wx^2}{4} = \frac{wl^2}{16}$$

$$\Rightarrow EI \left(\frac{dy}{dx} \right)_{\max} = - \frac{wl^2}{16}$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{\max} = - \frac{wl^2}{16EI} \quad \text{maximum slope}$$

Deflection :-

Integrate the eqⁿ (1) we get

$$EI \cdot \frac{dy}{dx} = \frac{wx^2}{4} + C_1$$

$$\Rightarrow \int EI \left(\frac{dy}{dx} \right) = \int \frac{wx^2}{4} + C_1$$

$$\Rightarrow EI \int \left(\frac{dy}{dx} \right) dx = \frac{w}{4} \int x^2 dx + \int C_1 dx$$

$$\Rightarrow EI \cdot y = \frac{w}{4} \times \frac{x^3}{3} + C_1 x + C_2$$

$$\Rightarrow EI y = \frac{wx^3}{12} - \frac{wl^2}{16} x + C_2 \quad \text{--- (3)}$$

Put the value C_1

Now find value of C_2 by using boundary condition if, $x=0$, $y=0$

in eqⁿ (3)

$$EI y = \frac{wx^3}{12} - \frac{wl^2}{16} x + C_2$$

$$\Rightarrow C_2 = 0$$

The value of C_2 in eqⁿ (2)

$$EIy = \frac{wx^3}{12} - \frac{wl^2}{16}x + C_2$$

$$\Rightarrow EIy = \frac{wx^3}{12} - \frac{wl^2}{16}x \quad \text{--- (4)}$$

This is deflection eqⁿ

But we calculate the maximum deflection using boundary condition.

When,

$$x = \frac{l}{2}, y = \text{maximum}$$

using the value in eqⁿ (4)

$$EIy = \frac{wx^3}{12} - \frac{wl^2}{16}x$$

$$\Rightarrow EIy_{\text{max}} = \frac{w}{12} \left(\frac{l}{2}\right)^3 - \frac{wl^2}{16} \left(\frac{l}{2}\right)$$

$$\Rightarrow EIy_{\text{max}} = \frac{wl^3}{12 \times 8} - \frac{wl^3}{32}$$

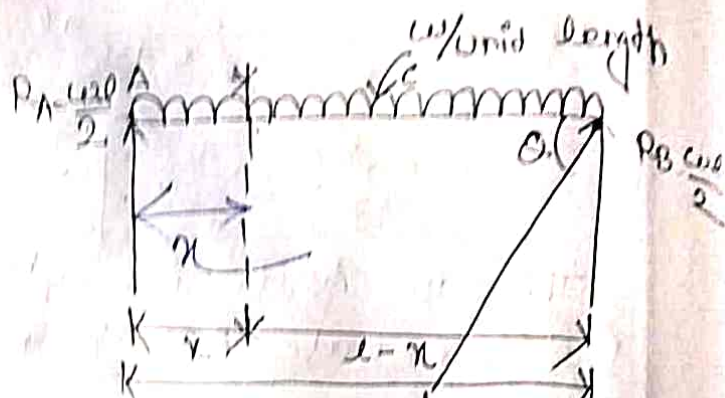
$$\Rightarrow EIy_{\text{max}} = \frac{wl^3}{96} - \frac{3wl^3}{96}$$

$$\Rightarrow EIy_{\text{max}} = \frac{wl^3 - 3wl^3}{96} = -\frac{2wl^3}{96}$$

$$\Rightarrow y_{\text{max}} = -\frac{wl^3}{48EI} \quad \text{maximum deflection}$$

This is the eqⁿ of maximum deflection.

Simply supported beam with U.D.L:



the load is symmetrically applied the reaction,

$$R_A = R_B = ?$$

$$R_A + R_B = WL$$

$$\rightarrow R_A + R_A = WL$$

$$\rightarrow 2R_A = WL$$

$$\rightarrow R_A = \frac{WL}{2} = R_B$$

Taking a section x-x at a distance x from A support.
The bending moment at the section is given by

$$M_{xx} = \frac{wl}{2} x - \frac{wx^2}{2}$$

$$wx \times \frac{x}{2}$$

We know,

$$M = EI \frac{d^2y}{dx^2}$$

$$\rightarrow EI \frac{d^2y}{dx^2} = \frac{wl}{2} x - \frac{wx^2}{2}$$

In integrating the above eqⁿ, we get.

$$EI \int \frac{d^2y}{dx^2} = \int \frac{wl}{2} x - \frac{wx^2}{2}$$

$$\rightarrow EI \cdot \frac{dy}{dx} = \frac{wl}{2} \cdot \frac{x^2}{2} - \frac{w}{2} \cdot \frac{x^3}{3} + C_1$$

$$\boxed{EI \cdot \frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1} \quad \text{--- (1)}$$

again integrate the above eqn we get

$$\Rightarrow \int EI \frac{dy}{dx} = \int \frac{wx^2}{4} - \frac{wx^3}{6} + C_1$$

$$\Rightarrow EI y = \frac{wL}{4} \cdot \frac{x^3}{3} - \frac{w}{6} \cdot \frac{x^4}{4} + C_1 x + C_2$$

$$\Rightarrow EI y = \frac{wLx^3}{12} - \frac{wx^4}{24} + C_1 x + C_2 \quad \text{--- (2)}$$

Where C_1 & C_2 are constant of integration obtained from boundary condition.

$$(x=0, y=0 \text{ \& } x=L \text{ \& } y=0) \quad x=0=L, y=0$$

By substituting $x=0, y=0$

In eqn (2)

$$EI y = \frac{wLx^3}{12} - \frac{wx^4}{24} + C_1 x + C_2$$

$$\therefore C_2 = 0$$

The value of $x=L, y=0$

in eqn (2)

$$EI y = \frac{wLx^3}{12} - \frac{wx^4}{24} + C_1 x + C_2$$

$$\Rightarrow 0 = \frac{wL^4}{12} - \frac{wL^4}{24} + C_1 L + 0$$

$$\Rightarrow LC_1 = \frac{wL^4}{24} - \frac{wL^4}{12}$$

$$\Rightarrow LC_1 = \frac{wL^4 - 2wL^4}{24}$$

$$\Rightarrow LC_1 = \frac{-wL^4}{24}$$

$$\Rightarrow C_1 = \frac{-wL^4}{24L} \Rightarrow C_1 = \frac{-wL^3}{24}$$

The value C_1 put in eqⁿ (1)

$$EI \frac{dy}{dx} = \frac{WLx^2}{4} - \frac{Wx^3}{6} + C_1$$

$$\Rightarrow EI \frac{dy}{dx} = \frac{WLx^2}{4} - \frac{Wx^3}{6} - \frac{WL^3}{24}$$

This is slope eqⁿ

but we calculate maximum slope
Put the boundary condition

$$x=0, \frac{dy}{dx} = \text{maximum}$$

$$EI \left(\frac{dy}{dx} \right)_{\text{max}} = \frac{WLx^2}{4} - \frac{Wx^3}{6} - \frac{WL^3}{24}$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{\text{max}} = - \frac{WL^3}{24EI}$$

maximum slope

then put the C_1 & C_2 in eqⁿ (2)

$$EI y = \frac{WLx^3}{12} - \frac{Wx^4}{24} - \frac{WL^3}{24} x + C_2$$

This is deflection eqⁿ

but we calculate maximum deflection put
boundary condition.

$$x = \frac{L}{2}, y = \text{maximum}$$

Put in eqⁿ (4)

$$\Rightarrow EI y = \frac{WLx^3}{12} - \frac{Wx^4}{24} - \frac{WL^3}{24} x + C_2$$

$$\Rightarrow EI y_{\text{max}} = \frac{WL}{12} \left(\frac{L}{2} \right)^3 - \frac{W}{24} \left(\frac{L}{2} \right)^4 - \frac{WL^3}{24} \left(\frac{L}{2} \right)$$

$$\Rightarrow EI y_{\text{max}} = \frac{WL^4}{12 \times 8} - \frac{W}{24} \times \frac{L^4}{16} - \frac{WL^4}{48}$$

$$\Rightarrow EI y_{\text{max}} = \frac{WL^4}{96} - \frac{WL^4}{384} - \frac{WL^4}{48}$$

$$\Rightarrow EI y_{max} = \frac{4WL^4 - WL^4 - 8WL^4}{384}$$

$$\Rightarrow y_{max} = - \frac{5WL^4}{384EI}$$

maximum deflection

Macaulay's Method

- It is also known as Double Integration method.
- In this method a section is taken far away from the left hand support.
- In this a systematic calculation of bending moment is done at the section.
- In Macaulay's method, we use the double integration concept to get slope and deflection respectively.

Problem

→ A cantilever beam 2m long carries a U.D.L of 10 kN/m over 1m portion from fixed end and a point load of 20 kN at free end. Calculate the maximum slope and deflection of the cantilever beam

by Macaulay's method.

Take $EI = 2 \times 10^9 \text{ kN-mm}^2$